Learning Calculus through PBL in an Industrial Engineering and Management Program - A Seven-Year Study

MARINEZ CARGNIN-STIELER  
Universidade do Estado de Mato Grosso  
Mato Grosso, Brasil  

M. TERESA MALHEIRO  
Centre of Mathematics / Department of Mathematics, University of Minho  
Guimarães, Portugal  

ANABELA C. ALVES  
RUI M. LIMA  
ALGORITMI Research Centre, School of Engineering, University of Minho  
Guimarães, Portugal  

AND  

MARCELO C. M. TEIXEIRA  
São Paulo State University (UNESP)  
Ilha Solteira–São Paulo. Brasil  

ABSTRACT

This paper aims to study ways of contributing to the development of Calculus through a Project-Based Learning (PBL) approach, which involves students in their first semester of the Integrated Master’s program in Industrial Engineering and Management (IEM) at the University of Minho. The study presents the analysis of forty-two interdisciplinary project reports developed by students over a seven-year period. The reports offer good evidence for the application of the requested Calculus learning outcomes, demonstrating that the application of Calculus content occurred during the development of the project. This finding means that PBL was successful as a truly active learning process for Calculus that involves students learning independently through the interdisciplinary resolution of problems related to their engineering professional profiles.

Key words: Project based learning, engineering mathematics, first year curriculum
INTRODUCTION

The curricular structure of the first years of an engineering degree includes a large number of courses that can be considered the basic sciences required for engineering (Lima et al., 2012). In general, engineering departments offer few courses in these first years of the program, and the interaction between the instructors from the basic sciences and the engineering departments is minimal (Kumar & Jalkio, 1999). There are a large number of studies investigating the processes of teaching and learning mathematics in engineering courses (e.g., Kumar & Jalkio, 1999; Lavesson, 2010; Engelbrecht, Bergsten & Kågesten, 2012; Diefes-Dux, Hjalmarson, & Zawojewski, 2013; Nathan, Srisurichan, Walkington, Wolfgram, Williams, & Alibali 2013; Dym, College, Agogino, Eris, Frey & Leifer, 2005) Nevertheless, these works do not present studies involving several years of project-based learning approaches, which would offer an understanding of how calculus learning outcomes can be developed through this active learning approach. Calculus courses are included in the basic sciences area, and it is considered a difficult subject by engineering students (Baracat, Witkowski & Cutri, 2012; Wrobel, Zeferino & Carneiro, 2013; Fragelli et al, 2013.). In general, engineering students take longer than expected to complete their degrees due to the difficulty in understanding the importance of the basic content in the first two years of graduation, which may be explained by pre-university difficulties in physics, chemistry and mathematics (Saeki & Imaizumi, 2013).

Students struggle to acquire mathematics skills, which are needed for their professional futures, and strategies for overcoming these difficulties are important. Biggs & Tang (2007, p. 24) believed that when students feel a need to understand a concept, they automatically try to focus on underlying meaning, main ideas, themes, principles, or successful applications. One way to create this type of inductive learning environment is based on Project-Based Learning (PBL) processes (Prince & Felder, 2006). In this type of learning environment, teams of students cooperate on projects and present a solution for an existing problem, which is characterized by a sufficient level of complexity for the level of competence that students are expected to develop in a predefined period (Graaff & Kolmos, 2003; Powell & Weenk 2003). Considering that PBL is an inductive learning solution that is widely proposed and implemented in engineering courses (Aquere, Mesquita, Lima, Monteiro & Zindel, 2012; Fernandes, Mesquita, Flores, & Lima, 2014; Lima, Silva, van Hattum-Janssen, Monteiro, & Souza, 2012; Kolmos, Graaff, & Du, 2009; Kolmos & Holgaard, 2010), it is important to reflect on the way students learn Calculus in engineering programs with Project-Based Learning.

The objective of this paper is to analyze the way Industrial Engineering and Management students learn Calculus through PBL — in this particular case, the way in which they use Calculus when prompted by the need to develop and complete the design of a product and a production system that involves mathematical content. This is the first Calculus course for these students. The
methodology used for the objective of this study was a document-based analysis of 42 reports of the projects developed by first semester teams of students in the Integrated Master’s program in Industrial Engineering and Management (IEM-IM) over a seven-year period. This study involved the identification of activities related to Calculus as described by the teams of students in the project reports to analyze the competences employed to apply the Calculus content.

This paper is divided into six sections. In the first section, the context of the study and objective is presented, and in the second section, a brief literature review is presented. The third section presents the study context, the fourth section explains the methodology used, the fifth presents the analysis, the sixth presents the discussion, and the seventh section concludes with the final considerations.

LITERATURE REVIEW

One of main issues of higher education research is the improvement of learning processes. Freire (1996) highlights curiosity as an essential component to be awakened by the teacher, thus inducing the student to question, understand and act. There are several methodologies that have been used to stimulate the curiosity of the student. Among these, project-based learning (PBL) instigates the students’ interest, arousing their curiosity in engaging the students in the learning process (Fernandes, Mesquita, Flores, & Lima, 2014). It is challenging to work from this perspective; one of the reasons is that both the teacher and students are challenged during the project. Working with project-based learning (PBL), the teacher can enrich the students’ work with questions that encourage learning and should remain aware of the possibilities for guiding learning in a critical and conscious way. In this sense, for Freire and Faundez (1985, p. 48), “[...] the first thing that the teacher should learn is to know how to question the students. When asked, know what the questions are that encourage us and stimulate society. Essential questions emerge from daily life because that is where the questions come from.” Awareness of these ideas can be used in the project and be translated as a problem to be solved by the students and guided by the teachers. Curiosity prompts questions and reflections on a situation, which may improve the students’ knowledge and future professional life. Exercising curiosity and questioning move students away from their comfort zones and encourages them to challenge themselves. Questioning exercises the right to receive answers and questions, participate in discussions and reflections and engage in new ways of thinking. Thus, the learning process may offer increasing value for the students and help them become skilled future professional.

Engaging students with real problems in their daily lives was proposed by William Kilpatrick (1871-1965) to improve their math classes. For this purpose, he developed the Project Method as
an activity that was designed to develop school activities through projects (Kilpatrick, 1918, 1921), which would motivate students to learn. The project-based methodology develops active learning skills, i.e., is a learning/teaching methodology centered in student learning. Active learning has been highlighted as teaching/learning processes that actively engage students in their activities and create opportunities for the students to reflect on their own learning processes (Bonwell & Eison 1991; Prince, 2004; Prince & Felder, 2006). Prince (2004) researched the most relevant forms of active learning for engineering courses and evaluates them positively, with special attention given to PBL because it positively influenced the students’ attitudes and their study habits, as well as improving their information retention and helping them develop critical thinking and problem-solving skills.

For Engineering Education, Powell and Weenk (2003,) proposed Project-Led Engineering Education (PLEE) as a teaching-learning approach that focused on team-based student activity relating to learning and solving large-scale, open-ended projects. During this process, students learn how to master the competencies specified in the curriculum within the context of the expected professional practice.

Learning through projects in engineering education enables the development of skills, abilities and attitudes and is an active methodology that is student-centered. This approach requires teachers to expand their understanding of the specific knowledge in their disciplines, maintain an open mind with regard to research, and promote group work. This is an important skill for teachers and students because working collaboratively in groups is a competence that facilitates insertion into the labor market; PBL allows them to develop this competence (Alves et al., 2012a; Aquere, Mesquita, Lima, Monteiro & Zindel, 2012). PBL, when adopted as a learning methodology, requires substantial changes in teaching and learning methods but has a significant impact on student learning and motivation (Alves et al. 2012b). PBL engage the students in learning; the greater the involvement is, the greater the interest is in learning (Biggs & Tang, 2007).

STUDY CONTEXT

European higher education system is based on 3 cycle of degrees, respectively Bachelor, Master and PhD programs. The bachelor in Portugal has 3 years and the master 2 years. An integrated Engineering master program is a 5 year program integrating bachelor and master degrees. This study was developed in the context of the first semester of the Integrated Master’s program in Industrial Engineering and Management (IEM-IM 11) at a university in Portugal, in which Project-Based Learning (PBL) has been adopted as a learning strategy in the last fifteen academic years (Lima et al., 2017). The project was developed by teams of students, and the project was one semester
long. A team of teachers from different schools and departments as well as tutors and educational researchers collaborated to introduce the PBL approach prior to the Bologna process (2004/2005) and perform the successive implementation of PBL to the present (Carvalho & Lima, 2006, Lima et al, 2007, Alves, Moreira & Sousa, 2007; Alves et al., 2012b). This implementation model in which students learn by working on projects has been the motivation for different studies in this context (Alves et al., 2012b; Fernandes et al., 2014; Alves et al., 2015). The instruction interdisciplinary team continued researching to improve the PBL model that was adopted to ensure effective learning for students, which is performed with the help of students who, at the end of semester, complete a survey and discuss the PBL learning process in focus groups. To support this process, documents had been designed to facilitate the progress and systematization of this methodology. These include the “project guide” and “tutor guide,” among others.

Industrial Engineering and Management Integrated Master’s program: first semester of the first year

The first semester of first year of any program is always an important and busy semester. Among all activities common to other freshman students (i.e., students at university for their first year to pursue higher education, typically with 17 to 19 years of age), this period is even more for the IEM-MI 11 students. The first week starts with the presentation of the project for the semester in the context of Project-Based Learning in the Integrated Project of Industrial Engineering and Management I (IPIEMI) course. In this presentation, the students are also grouped in teams, and tutors are assigned to these teams.

At this point, the students are equipped to start the pilot project in this first week. During this week, the stress is intense and the workload is heavy, but this is an experience that students do not forget and is useful for preparing them for the rest of the semester. This week concludes with a presentation from the teams to their colleagues and the coordination team (teachers, tutors and educational researchers).

Typically, six teams of students are formed with eight to nine students in each team. This number of members appears high, but as future engineers, they will be working in large teams. The insertion of the PBL course in the first semester of the first year is a fundamental factor for these students in the university context (Fernandes & Flores, 2011). This PBL course may help the students understand university education and offer responsibilities and awareness of transitioning from secondary school to higher education. The course may strengthen the professional and personal bonds between the students and teachers, which is a factor in the inclusion of freshman students in academic life. Additionally, learning is stimulated by conducting research.

The students’ teams developed their projects using the Industrial Engineering and Management contents to design a product and the production system of that product. To develop this project, the student teams must apply the content of project-supporting courses (PSC) (Powell & Weekk, 2003) of the first semester of the first year that are available. Thus, in general, the PSC for these
projects are Introduction to Industrial Engineering and Management (IIEM) and Algorithms and Programming (AP) from the School of Engineering and Chemistry (GQ), Calculus (CC) and Linear Algebra from the School of Sciences.

Each PSC teacher determines the content to apply in the project; for example, all IIEM content is applied in the project. The technical competences that each course conveys to students during the project execution are available in the project learning guide, which is considered a positive aspect of the study and knowledge of the content. The project learning guide is a document updated every year with a new project theme and description, instruction team, project milestones, learning outcomes of each curricular unit, and assessment model of each curricular unit and project.

Each project theme is briefly described in the project learning guide highlighting the importance of the theme followed by the objectives that students are expected to achieve. The project theme must be attractive to motivate the students. At the same time, must approach a contemporaneous theme connected to sustainability issues (Alves, Moreira, Leão, & Carvalho, 2017; Colombo, Moreira, & Alves, 2015; Moreira, Mesquita, & van Hattum-Janssen, 2011). In the first presentation of the project course to the students, the theme is contextualized with a video and/or real facts about the theme. Also, the project description and objectives are very broad and open to stimulate students’ creativity.

The project themes of the seven years analyzed in this work are presented in Table 1.

In the first two academic years, the objectives were too broad, as: 1) specify the product (fuel cell or water desalination system). This specification should be as rigorous and detailed as possible, in accordance with the competences defined for each of the subjects of the semester. There is no type of restriction on the type of fuel to be designed at the outset, however, each group should seek the endorsement of the coordination group for the product it intends to design; 2) specify a production system capable of producing the product defined above. The production system should be designed to respond to a given demand and justified by the team. Some results of a project from one team are in Figure 1, namely, the fuel cell structure and production system prototype.

<table>
<thead>
<tr>
<th>Project themes</th>
<th>Academic year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Draft of a fuel cell generation system</td>
<td>Year 1</td>
</tr>
<tr>
<td>2. Design of a water desalination system</td>
<td>Year 2</td>
</tr>
<tr>
<td>3. Production of batteries for an electric car: specification of the batteries and the production system</td>
<td>Year 3</td>
</tr>
<tr>
<td>4. Recycling organic waste to produce bio-alcohol</td>
<td>Year 4</td>
</tr>
<tr>
<td>5. Potable water production from air humidity: specifications of a portable device</td>
<td>Year 5</td>
</tr>
<tr>
<td>6. Collection and separation of oil spills at sea</td>
<td>Year 6</td>
</tr>
<tr>
<td>7. Design of a production system to disassemble electronics products</td>
<td>Year 7</td>
</tr>
</tbody>
</table>
The Figure 2 presents proposals of production systems, from two different students’ teams, to produce desalinized water in academic year 2.

The solutions provided by the students were so different that teachers felt the need to restrict the objectives, in a way that indicates better what they needed to do and to reduce the initial effort and somehow their first difficulty in choosing the product to design. Nevertheless, they continue to make diverse decisions (e.g. raw materials to use, product to produce) depending on the project theme. So, in the following cohorts the objectives were defined in a more restricted way, such as the one from academic year 3 (Figure 3):

1. Specification of the Battery System for an Electric Car: Specification of relevant vehicle parameters for sizing the propulsion system; Power of the battery system; Vehicle autonomy (specify schemes); loading time; dimensions; Weight; lifespan; environmental impact; limitations; technology employed;

![Figure 1. Results of a project team in academic year 1: a) fuel cell structure; b) a production system concept prototype.](image)

![Figure 2. Results of two projects in year 2: a) first draft of a water desalinization system b) an image of the production system.](image)
2. Specification of the production system for the production of batteries: Target market; monthly production; number of workers; Providers; management of supply of materials; production management; equipments; layout; costs, etc. Proposal of eco-sustainability of the production system (water, energy, materials, waste, etc.).

In the cohort of academic year 4 the objectives were “Present the different processes and raw materials used in the production of ethanol. Collect data on the use of organic waste to produce bio alcohol and produce bio alcohol in the laboratory. Show the pros and cons of current use of bio alcohol as fuel at impact level both locally and globally.” Here they could choose different raw-materials to produce the bio alcohol and the options varied from corn cane to grape cluck (Figure 4).

In academic year 5, students had to specify drinking water and present the processes / techniques of drinking water production by removal of moisture from the air. Also, they had to choose the process /
Learning Calculus through PBL in an Industrial Engineering and Management Program - A Seven-Year Study

Technique adopted and characterize relevant operational parameters and performance of the device. They had to produce drinking water in the laboratory and to present the advantages and disadvantages of extraction of air humidity from other production methods of drinking water (Figure 5).

“Collection and separation of oil spills at sea” was the project theme of academic year 6 and it was expected that teams should identify different physical and/or chemical methods used to clean off oil spills at sea. They had to discuss the advantages/disadvantages of each method and to propose an innovative solution for the collection and separation of crude oil from seawater. Assess the environmental impact of a spill considering the efficiency/inefficiency of cleaning it as a result of the method selected for waste. Here a myriad of solutions were also provided by the students as production systems, including a boat solution (Figure 6).

During the academic year 7 students had to design of a production system to disassemble electronics products according the following objectives: Identify the different chemical components used in the production of the product. Discuss how to treatment and recovery/re-use of each

![Figure 5. Results of a project team: a) the device designed; b) the layout and c) prototype of the production system.](image)

![Figure 6. Three different prototypes of production systems.](image)
component in order to reduce impact on man and environment. Assess the environmental impact of a wrong deposition on soil, water, or air, considering the efficiency / inefficiency of the current processes and the solution proposed under this project. The raw-material (product to dismantle) was also very diverse from hairdryers, PlayStation, television, phone, printer, car multimedia systems. Figure 7 represents three examples of these products. Because an electronic product includes so many raw-materials they had to make the decision on what they should focus (e.g. metal, plastic, etc).

**Calculus in the project**

Calculus is among the curricular units of the first semester of the first year that support the project development. Calculus content was introduced in all projects analyzed in this study. This course is offered by the Mathematics Department in the School of Sciences, which assigns a teacher to the course and assigns another teacher to support the project in smaller sessions.

The project theme is defined by all the teachers assigned to each course, and the integration of the content is defined by each teacher. During the project, with the support of the teachers, students understand the reasons that Calculus competence is important for the project and for their futures as engineers. The technical content of Calculus is basically the same throughout the years, with some minor changes occurring due to the developed pathway and interdisciplinary maturity of project and their participants. In the context of the project, the students must apply this content to develop and optimize certain functions that are selected by them and are suitable for the problem they need to solve.

To guide the student teams during the write-up period, a checklist of the content of each course (related to the competences that were to be developed) was created by the coordination team to introduce the reports. This checklist serves two purposes: the one purpose already mentioned is to guide the students and serve as a guide for the assessment of other reports by different teams and to give feedback on these reports. This peer assessment and feedback assigns additional responsibility to the teams while also giving them the opportunity to read different reports. The coordination team believed that this practice is beneficial for the students, and the students agree with this. Thus, to create an increased understanding on the way in which the technical aspects of Calculus can be integrated into their projects, the students receive the list shown in Table 2.
Table 2. Technical aspects to consider for Calculus (in a specific project year).

<table>
<thead>
<tr>
<th>Technical issues of Calculus to consider in the reports</th>
</tr>
</thead>
<tbody>
<tr>
<td>To explain and justify the need to determine the areas/volumes and extremes of the real functions of one variable;</td>
</tr>
<tr>
<td>To write a short theoretical introduction presenting the use of formulas (formulas to calculate the area of a two-dimensional region, the surface area, the volume of a solid of revolution, and the arc length);</td>
</tr>
<tr>
<td>To discern all the components that are to be determined (rather than showing sketches, when applicable);</td>
</tr>
<tr>
<td>To properly define the necessary formulas to calculate the volumes and arc length or surface area of the revolution;</td>
</tr>
<tr>
<td>To correctly calculate the integrals and other quantities as well as the extremes of the real functions of one variable.</td>
</tr>
</tbody>
</table>

Table 3 presents the list of six learning outcomes (LO) defined for the Calculus course. From these six learning outcomes, outcomes 1 to 4 are expected to be integrated into the project.

Table 3. Calculus learning outcomes (LO) for the project.

<table>
<thead>
<tr>
<th>LO Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>To analyze the real function of one real variable: the monotony, continuity and differentiability in its domain;</td>
</tr>
<tr>
<td>2</td>
<td>To apply the derivatives of problems;</td>
</tr>
<tr>
<td>3</td>
<td>To understand the notion of a definite integral of a bounded real function;</td>
</tr>
<tr>
<td>4</td>
<td>To use integrals in several applications;</td>
</tr>
<tr>
<td>5</td>
<td>To study the convergence of a numerical series;</td>
</tr>
<tr>
<td>6</td>
<td>To define a power series and determine its convergence interval.</td>
</tr>
</tbody>
</table>

Source: Adapted from the project guide of IEM11.

RESEARCH METHODOLOGY

Project-based learning, as described previously in this work, is based on an open problem that is addressed by teams of students to simultaneously serve as a means of the application of content and as a driver for extending the understanding of that same content. In the case study presented in this work, every year, the students examined a different problem and were required to apply the content, including Calculus, in a different way. This means that the application of Calculus will vary between years, mainly due to different projects’ objectives and the different ways that the teams of students used the specific content. Considering that the main objective of this paper is to understand the way Industrial Engineering and Management students learn and apply Calculus through PBL, a thorough analysis of the reports written by the students over several years can offer an important contribution to this objective.
Document analysis is a technique that is particularly relevant for the identification of relevant information, facts and evidence in documents in light of the research questions. When considering the research questions, the goal of the document analysis will allow for the understanding of a particular phenomenon and produce a set of inferences that constitute a significant contribution to the research (Coutinho, 2013). Thus, in this work, beyond the general objective above, the researchers intend to answer to the following questions:

- What were the Calculus learning outcomes introduced in the reports?
- Which Calculus competences did the students most often develop, based on the results presented in the project reports?

To answer to these questions, this paper reflects upon the document analysis of 42 reports produced by PBL student teams over a seven-year period. The researchers conducted several observations of these reports, which were developed during this time frame in a retrospective longitudinal case study (Saunders, Lewis, & Thornhill, 2009). The main goal was to analyze the application of Calculus in the interdisciplinary project reports that were created by the teams of students to infer the competences learned in the context of this course. Each semester, six teams were formed over seven years, resulting in the 42 reports analyzed in this longitudinal study.

First, a general analysis of the reports was performed. The reports were submitted in the 17th week of each semester, and it was observed that all teams’ reports were a maximum of 60 pages, excluding the annex. All texts were produced with rigor and clarity. They present an abstract, cross references and list of references according to a referencing system. All 42 reports that were analyzed were different with regard to writing and research. It was also possible to infer the knowledge acquired by the students during the semester.

Second, an analysis of the content was performed that, according to Bardin (2002), comprises a set of technical analyses that require a description of procedures and the systematic and objective description of the content indicators that allow the inference of relevant knowledge. The content analysis was based on the thorough reading of all 42 reports, checking if it show evidences of development of any of the Calculus learning outcomes (LO) for the project (Table 3 above). Each of these evidences was classified as being related to specific LOs. After the classification phase, the number of reports, the number of teams and the number of evidences per LO per year was computed. In summary, each LO presented in Table 3 above was analyzed in detail based on each of the 42 reports to identify whether the Calculus LOs were understood by the students. Additionally, a complementary analysis was made in order to try to develop insights on the way some teams applied Calculus content. This part of the analysis allows to develop some insights on the way students have been applying the Calculus content and have been creating interdisciplinary links with the project objectives. Thus, it allowed to analyze the most important
results that were found in the reports regarding non-conventional creative strategies, critical thinking, and self-evaluation.

**DOCUMENT ANALYSIS**

At the end of the semester, the student teams delivered the final reports presenting their cumulative work. These reports were assessed by the teacher team.

In this section, we intend to answer to our research questions. In the first subsection, we identify the Calculus Learning Outcomes (LO) presented in the reports, and we will ascertain which of these outcomes were more thoroughly developed by the students. In the second subsection, we analyze the subsequent results presented in these reports.

**LO introduced in the reports**

Each LO presented in Table 3 above was analyzed in detail based on each of the 42 reports to identify whether the Calculus LOs were understood by the students. We observed that not only they were present, but the Calculus content was applied in several cases of the 42 reports in the study.

In Table 4, we can see the number of reports in each year that presented sufficient knowledge that allowed us to determine the LOs, presented in Table 3 above, that were achieved.

As can be observed in Table 4, most of the teams were able to introduce 4 out of the 6 LOs that were expected in the developed projects — the competences related to **integrals application**

<table>
<thead>
<tr>
<th>LO number</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
<th>Year 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>–</td>
<td>1</td>
<td>–</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>+</td>
<td>–</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>–</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>+</td>
<td>–</td>
<td>14</td>
<td>4</td>
<td>9</td>
<td>–</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: *Number of teams that introduced references in the final report. +Total number of references cited by the teams in the final report.
were required and evaluated by the Calculus teacher. We also analyzed the way in which the other Calculus LOs were introduced and used in the projects that are described next. The Learning Outcome 1 states that, at the end of the course, students should be able “to analyze a real function of one real variable: monotony, continuity and differentiability in its domain”, was applied in almost all of the reports (80.9%) following a homogeneous distribution over the years. This content is taught in secondary school, and they were used in the projects to refine the visualization of competence with integrals.

The Learning Outcome 2, is defined as students should be able “to apply derivatives in problems”, was evident in 14 reports (33.3%). In the first three years, only 2 reports each year achieved this LO, and all the reports in the final year (6 reports, almost half of the 14 reports) sufficiently achieved this LO. In the final year, the Calculus support teacher adopted a different attitude and recognized the struggle the students experienced in apply the mathematical principles; he decided to present the students with two specific open problems (Cargnin-Stieler, Lima, Alves & Teixeira, 2013) that resulted in applications of derivatives.

The Learning Outcome 3 was described as students should be able “to understand the notion of a definite integral of a bounded real function,” and the Learning Outcome 4, defined as students should be able “to use integrals in several applications,” were required and appeared to be sufficiently attained in all reports (100% of the reports). This content is only taught at the university level in the European Union. An example of how students include these LO is presented in the following quotation from one report:

*The aim is to study solids in order to choose the best material in which to store the components that will be sold and recycled. Thus, we use these equations to determining the volume of a solid revolution and surface area. (Report 37, p. 27)*

\[
V = \int_0^1 \pi [f(x)]^2 dx \\
A = \int_0^1 2\pi f(x) dx + \pi [f(x0)]^2 + \pi [f(x1)]^2
\]

The Learning Outcome 5 that was presented as students should be able to “study the convergence of a numerical series” and Learning Outcome 6, students should be able to “to define a power series and determine its convergence interval,” were not applied. The related content is not required in the project and is normally taught at the end of the semester. This content requires a deep knowledge of other theories for application in complex real situations. One application of power series can be found in electricity systems (e.g., Rodrigues, Crepe, Porto, Ulson, Serni, 2010; Nascimento, 2004).
Upon observing the research development and mathematical argumentation quality, the teams understood the requirement to apply the Calculus content and solve the proposed problem, as evidenced by the following quotation:

*Once the ratio for the chosen revolution solid is obtained, we propose to compare it with the optimal solid obtained previously because the cylinder had a ratio that was approximately equal to 5.54. Our revolution solid is inferior in this aspect, which implies that the cylinder obtained is the best among all those considered. (Report 39, p. 59)*

It is possible to conclude that for the same volume, the cylinder has a smaller surface area than the solid of the revolution created by the group. Thus, it is helpful to use the cylinder as a container for storage.

*Thus, the cylinder dimensions given a volume = 1m³ are the following:*

\[
h = \frac{V}{\pi r^2} = \frac{1}{\pi \left(\frac{V}{2\pi}\right)^{\frac{3}{2}}} = 1.08 \text{ m}
\]

\[
r = \sqrt[3]{\frac{V}{2\pi}} = \sqrt[3]{\frac{1}{2\pi}} = 0.54 \text{ m}
\]

*It follows that the ideal cylinder has h = 2r. (Report 38, p. 53)*

It was also possible to see that the Calculus knowledge resulted in greater objectivity in the proposals presented and in decision-making, as presented in the following quotation:

*The definition of the container shape is important, not only in terms of the material costs but also for the space occupied in the company facilities. The choice of the optimal solid to use for the containers’ shape is therefore very important. (Report 39, p. 51)*

### Some complementary results of the way students applied Calculus

Learning Outcomes 3 and 4 were the LOs with large number best developed of the 42 reports, and their application produced a significant amount of visible results by facilitating the creation and use of geometric solids in the construction of the product prototype to respond to the requirements of the project. Additionally, it was considered important to explore the way students applied...
Calculus content. Despite the fact this part of the analysis is not based on a thorough classification and quantification of the way students applied Calculus content, it allowed to develop some insights on the way they have been applying the Calculus content and have been creating interdisciplinary links with the project objectives. Thus, although the evidences presented in this section are scarce, they show some useful information on the way students developed arguments and applications of the Calculus content.

**A. Non-conventional creative strategies**

Evidence of the non-conventional forms and strategies that were adopted by the teams demonstrated creativity and thought with regard to the application of math, which show the learning and confidence of these students regarding Calculus content. In this sense, one of the teams in the project, for which the objective was to produce potable water from humidity (academic year five), created a portable device image that, according to some arguments, may be successful as entrepreneurial innovation (Figure 8). The students describe this image as follows:

*The company [the student team] thought of an innovation image that is suitable for all client tastes. It was decided that this would be in the form of a jar. This is considered a futuristic and aesthetic vision because this form does not exist in any form in the market, can fit into any area of a house, and is easy to transport.* (Report 28, p. 14)

They continue,

(...*) the device will have a height of 57 cm, a radius of 14 cm and a volume (v1) of 20 liters. However, through the knowledge acquired in the Calculus classes, it was possible to

![Figure 8. Design of the portable device that would produce potable water from humidity. Source: Report 28, p. 15.](image)
determine the functions corresponding to the solidity of the revolution, the volume of the
device, and the surface area of the revolutions with the help of integrals (Report 28, p. 15)

B. Critical thinking

The students were able to verify that the identified answer was not the expected one, so they
sought assistance after validating the results; they concluded this issue was not the results of prob-
lems in the study. An important aspect of this PBL study is comparing the expected results with
those obtained. This situation shows that the students used Calculus techniques to solve real world
problems and employed critical thinking. The text shows this concern:

The integral calculation presented in the preliminary report was not the expected result
because the values were too large. Thus, we concluded that were some errors in the integral
development. The same occurred for the revolution surface area. To solve this problem, the
team consulted the Calculus teacher and reformulated the graphic functions to obtain the
correct calculations (Report 28, p. 15)

Beyond the notion of the expected result and the students’ persistence, one can understand the
relevance of this type of learning to the learner and the proximity of the students to the teacher and/or tutor.

The software use facilitated the Calculus application in the projects and was applied mainly when
developing integrals and calculating areas and volumes. Meanwhile, with regard to the teams’ creativ-
ity in visualizing non-conventional solids, the studies extended beyond the acquired knowledge. In the
reports, difficulties encountered by the teams were observed when they were faced with complicated
integrals that they did not solve without specific programs. However, this situation was not a demoti-
vating factor that was an obstacle to the creativity of the solid in the study. In this sense, the authors
recognized that the students advanced in their research because they used methods and programs to
obtain the expected results. It is also important that the students recognized that their knowledge was
not sufficient. They needed to seek other tools, after which they became aware of numerical software
that could help them perform difficult calculations. Among the final remarks of report 29, the students
describe the use of a program to solve the integrals using the numerical approach method. They wrote:

In the context of Calculus, the task was to calculate the device's total volume and the
respective surface area. The physical shape of the device was given through a function that
was obtained by trial. The graph of the function of the rotation of the x-axis in a Cartesian
reference resulted in the shape of the device in question. Thus, to determine all data for the
device, integrals were applied. Some integrals cannot be obtained at this time, and the team
used the “Scientific Workplace” program, which allowed the team to identify the solution of the required integrals (Report 29, p. 54)

The students did not abandon their efforts when they faced difficulties, and they did not redesign the prototype; instead, they sought alternatives to solve the Calculus block and keep the prototype. Figures 9a and 9b show the visual representation of the device designed in this report and the graphic representation of the functions that created this device.

The students preferred to keep the prototype and further analyze the calculations. They were able to obtain a representation that was similar to the regular shape because the initial proposal had requirements beyond the integral calculation to find the volumes and areas of known shapes or those that are normally described in the books.

C. Self-evaluation

A diverse array of techniques can stimulate interest and objects of study, mainly at the undergraduate level, by serving as a learning facilitator. Yuen, Saygin, Shipley, Wan & Akopian (2012) believed that such technologies could be used to support instruction while motivating and involving the students in the learning. This affirmation is easily seen in the report analysis.

In report 35, the project — from the academic year six — was the collection and separation of oil spilled at sea, in which a team of students created a ship with decantation towers. After designing and describing the production system, they present all calculations of three geometric structures: the cylindrical towers, the dome (ellipse) and the cone (the tower bottleneck). They reported the following:

The Calculus application was an essential element for this topic because, through integrals, we obtained the volume calculation and eventually the areas of the solids that had unusual...
shapes that would have otherwise been impossible. Thus, with the knowledge acquired from Calculus, we understood that the suggested proportions for the objects of our production system are the most suitable for attaining the efficiency level and required quality. (report 35, p. 29)

Among the reports, it was possible to experience real-world situations where content from Calculus can be applied. In traditional classes, it is responsibility of the teacher to justify the importance of a calculation that is sometimes not clear to the students. For example, the following quotation from one report shows the importance of Calculus for a project in the perspective of the students:

Having information about the method to obtain water and its quality, we proceed to the physical conception of the device. For this, the acquired knowledge in the Calculus course was very useful. We designed a device shape and, through integration, its dimensions and volume were calculated. (Report 28, p. 60)

Beyond the LOs of the Calculus course, the students also used other mathematical knowledge and reasoning to determine the number of workers, cost minimization, profit maximization, forecasting and average estimations, which allowed them to propose economically viable solutions, clarify decision-making, and evaluate the production system. At the same time, the proposed solutions must adhere to ethical, moral and legal regulations. In this sense, it was possible to clearly observe the integration of the courses and the concretization of the interdisciplinary project.

Beyond the competence analysis, the difference between the number of bibliographic references referring to the Calculus course in each report was recognized. It is important to mention that among the bibliographic references, six Calculus books with the correct citations were included with the report to clarify the development of the study. The learning extension was observed and noteworthy for the teams that sought references beyond the lecture notes.

**DISCUSSION**

In a deeper analysis of the reports, it was possible to understand the evaluations and reflections of the activities performed in the projects, the application of the knowledge acquired in the courses, and the research performed. Through the conclusions of the reports, it was possible to follow the
teams’ work, some of whom reported that difficulties emerged during the semester. This is expressed in the following quotation from one report:

\[
\text{The achievement of a project like IPIEM involves difficulties that are not easy to overcome, which become significant obstacles. In this way, significant effort was necessary from all team members, and it was essential that everyone knew to work in teams to achieve the objectives proposed. It is necessary to interrelate all the knowledge that was acquired in the various courses and perform research to achieve the result required (report 33, p. 46)}
\]

The students understand that the tutor determines the success of the team during the semester and is critical for overcoming the initial difficulties. This is in line with the research of Alves et al. (2007, 2010) in the context of these projects. In the same reports, the difficulties encountered were described, in addition to the value of the work performed and the achievements made. Among the remarks, the following is highlighted:

\[
\text{The team concluded that this project had an important role in the consolidation of the content taught in the various courses in addition to stimulating the research. For the future, the team believed that the project was an important experience to remember because it revealed aspects of our future profession and demonstrated the importance of teamwork (Report 33, p. 46)}
\]

\[
\text{This project was a major milestone at the beginning of our path as university students. Learning was focused on the student, not on the teacher, as established by one of the objectives of this project — our autonomy. It was also possible to develop our teamwork skills, manage conflict, develop critical thinking and decision-making skills, which serve to foster transversal competences. (Report 36, p. 4)}
\]

It was observed that the teams experienced and reported the facts through their capacity to understand the learning acquired during the semester. According to Fernandes, Mesquita, Flores, and Lima, (2014) the interdisciplinary projects provided relevant experience that was capable of enabling learning construction because the students constructed meaning between the acquired information and learning experiences. The authors realize that for the students, working on an interdisciplinary project provides the construction of meaning for the Calculus content.
Beyond the application of Calculus, the research performed reported the importance of this methodology. The conclusions are shared below:

*The development of this project implied the adoption of another learning type, a new method of learning in which the acquisition of new knowledge implied greater effort from our side as well the development of correct methods with the aim of achieving efficient solutions for the problems proposed (Report 36, p. 4).*

*The project, over time, offered a fantastic experience for all team members, which contributed to the development of all members as students and individuals. With regard to personal development, the relationship between students and teachers was very positive, which is essential not only to team members but also to colleagues of the same year, program, academic community and society. (Report 26)*

To offer students effective higher-level education in engineering programs, the faculty must implement engineering-related work that allows the students to apply their knowledge and develop their interests (Yuen, Saygin, Shipley, Wan & Akopian, 2012). This can be accomplished by allowing the students to work on complex projects with their peers and work with engaged teachers. The students began with a simulated open problem that is realistic and relevant to their professional context. They then had to cope with uncertainty and propose a simulated production system adapted to their level of knowledge. With this, they could then apply the content covered in class or research further and produce a report with accuracy and clarity.

It was concluded that the projects had improved with the application of the Calculus; the analysis of the reports show that the application of Calculus is possible in the context of the chosen themes. The content addressed depends more on the creativity and efforts of the students than the demands and rigor in ensuring that all the topics are covered in the project. By participating in interdisciplinary projects, the Calculus teacher intended for the students to understand the way in which the content was covered in the class, including mathematical formulas, and translate and model certain situations, after which the goal was successfully achieved. This discussion occurs from the perspective of the student on the applicability of Calculus; for an academic context, knowledge is used to simulate a situation in the context of their future profession (Alves, Moreira & Sousa, 2007).

The authors understand that the application of Calculus is based on three components: the students, teachers and the theme. From the students, creativity, knowledge and the pleasure of studying are expected. From the teachers, competence in stimulating the students and teaching the
students the content and research resources are expected. Some project themes may have been more attractive and simpler for Calculus application.

The importance of the interdisciplinary project for Calculus was observed from the reports through the content consolidation obtained in the classes. Through the report analysis, the authors also observed the application of antiderivation and use of integrals to determine two-dimensional areas, volumes and surface areas of solids of revolution. Thus, it is possible to conclude that Calculus-related technical competences were achieved successfully. In the conclusion of a project report, a team analyzed the knowledge process acquisition and highlighted the following:

*After all the work, effort and commitment dedicated to this project during the semester, team 5 is proud of the results achieved and believe that we have attained all objectives. This project contributed significantly to the growth of each member of the team and helped facilitate the creation of new skills due to the project concretization and teamwork* (Report 29, p. 54)

From the general analysis of the reports, the authors conclude that these results have gone beyond the expected goals of students in their first year of a higher education engineering course.

**FINAL REMARKS**

This paper studied the activities performed by students teams in a Calculus course to analyze the acquisition of competences, as observed in the reports of interdisciplinary projects during the first semester of the first year of the Integrated Master's program in Industrial Engineering and Management over a seven-year period.

In the light of the analysis issues, it is possible to conclude that the competences demonstrated in the reports overcome the expectations of the authors because all reports demonstrated good examples of the application of the requested competences. In relation to the types of Calculus knowledge and competences that were introduced in the reports, the competence “analyze a real function of one real variable: monotony, continuity and differentiability in its domain” was the best developed in most of the reports. The application of “derivatives in problems” was fully applied only in the reports from the year seven. The most clear and intense evidence of all 42 reports examined is the Calculus competences that students developed the most in this Project-Based Learning process, was the “notion of definite integral of a bounded real function” and “integrals in several applications.”
Thus, it is possible to conclude that Calculus applicability occurred by means of an inductive search for solutions made by students during the development of the project, which means that PBL succeeded for a Calculus course in serving as a truly active learning process that involves the students in their own learning through the interdisciplinary resolution of the problems related to the engineering profession. This type of learning approach is in accordance with the current curricular recommendations (SEFI, 2002) that incorporate a wider view of mathematics for engineers: “Mathematical competence is the ability to understand, judge, perform and use mathematical concepts in relevant contexts and situations, which certainly is the predominant goal of mathematical education for engineers” (SEFI, 2013, p 7).

ACKNOWLEDGMENT

This work has been supported by FCT – Fundação para a Ciência e Tecnologia (Portugal) within the Project Scope UID/CEC/00319/2019, by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) (001) and by National Council for Scientific and Technological Development (CNPq) - Brazil (309872/2018-9).

REFERENCES


Learning Calculus through PBL in an Industrial Engineering and Management Program - A Seven-Year Study

Research Centre in Education (CiEd) University of Minho and Department of Production and Systems School of Engineering of University of Minho.


Learning Calculus through PBL in an Industrial Engineering and Management Program - A Seven-Year Study


**AUTHORS**

**Marinez Cargnin-Stieler** holds a degree in mathematics from the Federal University Santa Maria (UFSM) in 1987. Masters in Mathematics Teaching from Universidade Franciscana (UFN) completed in 2007. She concluded her Doctorate in Electrical Engineering at UNESP – Univ. Estadual Paulista. FEIS - Faculdade de Engenharia de Ilha Solteira in Ilha Solteira - SP, Brazil in 2014 now, she is a Senior Professor of UNEMAT - Universidade do Estado de Mato Grosso, Cuts – Campus Universitário Professor Eugênio Carlos Stieler/Tangará da Serra - MT, Brazil. In 2004, she taught there and was Campus Coordinator. She was Coordinator (2014-2017) of the Engineering at Cuts-UNEMAT. She has published over 40 papers and she is author and co-author of eight books chapters and one edited books. Her interests include statistics, teacher education and engineering education.

**M. T. Malheiro** is an assistant professor in Department of Mathematics and Applications of Science School, at University of Minho. She holds a Degree in Mathematics Education and a PhD in Science from University of Minho. Her research is inserted in Functional Analysis and Operator Theory. She is also interested in Engineering Education and has collaborated in interdisciplinary PBL experiences at University of Minho.
Learning Calculus through PBL in an Industrial Engineering and Management Program - A Seven-Year Study

Anabela Carvalho Alves is Assistant Professor in the Department of Production and Systems - School of Engineering - University of Minho. She is member of ALGORITMI Center. She holds a PhD in Production and Systems Engineering. Her main research interests are: Design and Operation of Production Systems; Lean Production; Industrial Engineering and Management Education with focus on Lean Education and Project-Based Learning. She has published over 130 papers and she is author and co-author of two books and two edited books. She supervised one PhD and over 50 master dissertations in Industrial Engineering and Management. Currently, she is supervising five PhD students and more than 20 master dissertations. She is member of the board of the Master in Industrial Engineering. She is a permanent member of the Scientific and Organizing Committee of the International Symposium on Project Approaches in Engineering Education (PAEE). She was involved in three R&D projects developed in partnership with Bosch company. ORCID: http://orcid.org/0000-0002-2926-4187

Rui M. Lima is an Associate Professor of the Department of Production and Systems and Integrated Member of the ALGORITMI Centre - Industrial Engineering and Management Research Line - of the School of Engineering of University of Minho, Portugal. His main research interests are related to Industrial Engineering and Management fields: Production Management; Lean and Agile Project Management; Lean Services; Lean Healthcare; Engineering Education; Project-Based Learning (PBL); University-Business Cooperation (UBC). He is involved in international projects with universities of eight different countries from Europe, Asia and South America. He has more than 150 peer reviewed publications in journals, conferences or book chapters, and he acted as invited editor of four special issues in indexed journals. Rui is one of the chairs of the “International Symposium on Project Approaches in Engineering Education” (PAEE) since 2009. He is the current president of the PAEE association and the former chair of the steering committee of the Active Learning in Engineering Education - ALE network.
Marcelo Carvalho Minhoto Teixeira holds a degree from the Escola de Engenharia de Lins (EEL), Brazil, completed in 1979. He has a Masters degree from Universidade Federal do Rio de Janeiro (UFRJ-COPPE), Brazil, completed in 1982. He concluded his Doctorate at the Pontifícia Universidade Católica do Rio de Janeiro (PUC/RJ), Brazil, in 1989, in Electrical Engineering. In 1982, he joined the Department of Electrical Engineering, FEIS, UNESP - Univ Estadual Paulista, Ilha Solteira - SP, Brazil, where he currently holds a Professorship. In 1996 and 1997, he was Visiting Professor at the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN, United States. He was Coordinator (2005-2007) and vice Coordinator (2000 to 2005 and 2007 to 2010) of the Postgraduate Program in Electrical Engineering at FEIS-UNESP. He has been a member of the Brazilian Evaluation Committee of Postgraduate Courses - IV Engineering CAPES since 2009. Additionally, he was the Coordinator (1991-1993) and vice Coordinator (1989 to 1991 and from 1993 to 1995) of the Undergraduate Program in Electrical Engineering at FEIS-UNESP. Dr. Teixeira is an Associate Editor for the IEEE TRANSACTIONS ON FUZZY SYSTEMS since 2016. His interests include control theory and applications, education in control, neural networks, variable-structure systems, linear-matrix-inequality, fuzzy systems, and switched systems.