Project Overview:

An analysis of the behavior of the bell tower under harmonic loads is desired. As a variety of rhythms and sounds will be generated, a range of forcing function magnitudes and frequencies will be considered. Additionally, as concrete is likely to crack if enough tension is present, an evaluation of the system utilizing cracked section properties will also be conducted.

The response properties of interest are:

- peak *lateral* displacement at the top of the tower in the plane of the drawing.
- Shear force and stress
- Bending moment and flexural stress

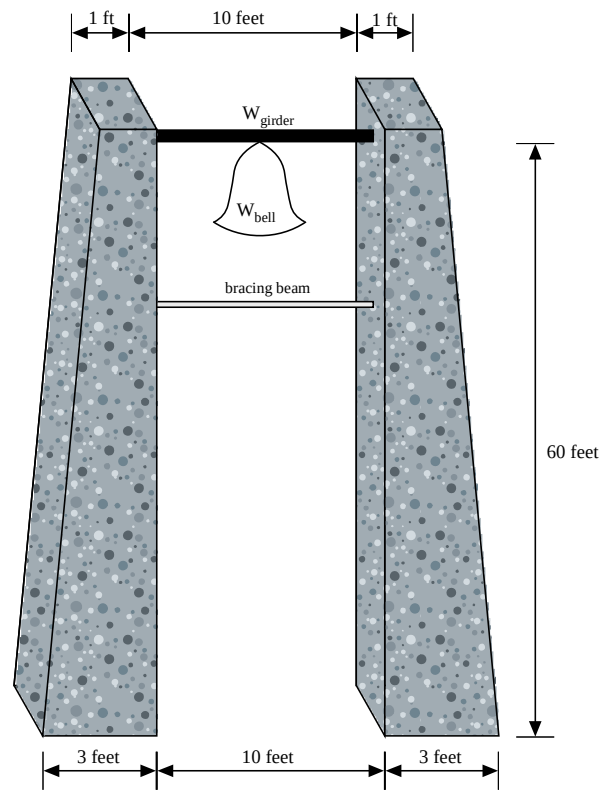

Figure 1: Bell Tower Schematic

Project Goal and Objectives:

1. Determine the dynamic response of the tower to a harmonic load
   - Range of force magnitudes
   - Range of frequencies
2. Determine the lateral displacement response
   - Total response
   - Steady-state response only
3. Determine the combined static and dynamic forces/moments at the base of the tower as well as corresponding stresses.

Excitation Description:

The response of the system will be performed using a harmonic load of the form:

$$F(t) = F_0 \sin(\bar{\omega} t)$$

The magnitude of the force function, F_o, is specified as a fraction of the weight of the bell, and the possible values to be considered are:

$$F_0 = \left[\frac{1}{25} \quad \frac{1}{20} \quad \frac{1}{15} \quad \frac{1}{10} \quad \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \right] * w_{bell}$$

The forcing function frequency ranges between 10 and 100 rpms, with the following values to be explicitly considered:

Frequency (rpm)	100	75	60	50	25	20	15	10
$\bar{\omega}$ (rad/sec)	10.472	7.854	6.2832	5.2360	2.6180	2.0944	1.5708	1.0472

Part 2 -- Detailed Tasks:

Part 2.1 – Modeling Refinement

Your team has already completed the basic modeling of the system. Depending on the feedback received, you may need to update your model to better capture the system. This first model is to be referred to as the *linear uncracked model of the tower*.

Additionally, a second model is to be considered where some of the column is assumed to crack under tensile stress. The code allows for evaluation of lateral deflections utilizing 50% of the uncracked section properties. So determine a new model, *linear cracked model of the tower*, by reducing the effective stiffness for the columns by 50%.

Part 2.2 – Dynamic Equation and Properties for BOTH models

a) Determine the equation of motion for the system using the ***relative*** horizontal motion of the top of the towers to the ground as your degree of freedom.

b) Determine the undamped natural frequency, ω, and the corresponding undamped period, *T*, for this system.

c) Some energy loss will occur in this system due to cracking of the concrete, friction at connections, etc… To account for the energy loss, a damping ratio of 5 % is assumed. Determine the corresponding damped frequency for the system.

Part 2.3 – Displacement and Force Response

a) Determine the dynamic response of your system (displacement and rotation/slope at tip). Take your initial conditions to be zero for relative lateral motion. (Note: how can you relate the rotation/slope to strains? Will need that for the next part.)

b) Develop a plot for the time history response (displacement vs. time) for each model. Comment/compare the plots obtained. Some things to consider: the change in amplitude of vibration, the change in frequencies (both model and force function). You must determine how to best present the comparisons – it will NOT be to just include a plot for each individual simulation you have done.

c) Utilizing the response results, determine the reactions (horizontal, vertical and moment) at the base of the columns due to dynamic behavior only for the system. Again, you must do this for both models and the various combinations of frequency and force magnitude. Determine how to best show the results (place peak values in a plot? In a table?).

d) As the columns must also carry static loads, determine the reactions (horizontal, vertical and moment) at the base of the columns due to the *combined* static and dynamic loads for the system. Again, you must do this for both models and the various combinations of frequency and force magnitude. Determine how to best show the results (place peak values in a plot? In a table?).

e) Based on the previous results, determine the peak shear and flexural stresses of the column for both models and the various combinations of frequency and force magnitude.

Part 2.4 – System evaluation and recommendation

Based on the previous results, comment on how well you believe the models capture the response of the system. Additionally, consider how well the system behaves under the loads: Does it seem sufficient? Overdesigned? Underdesigned? What changes would you recommend to the system and why?

Keep in mind the following:

- Concrete can be assumed to crack in tension when tensile flexural stresses reach: $f_r = 7.5\sqrt{f_c'}$, where f_c' must be entered in psi.

- Concrete is deemed to fail when the strain in compression reaches 0.003

- Ultimate shear force capacity can be found by $2bd\sqrt{f_c'}$ -- for simplification, take:

  b = width of column (1 ft)
  h = depth of cross-section (varies from 1 ft at top to 3 ft at bottom)
  $d \approx h - 3$ inches

Note: concrete behaves nonlinearly. While a linear analysis using cracked section properties is allowed by code, the capacity is determined accounting for the nonlinear behavior and knowing the steel reinforcement. For the purposes of this project, assume that a first-pass estimate utilizing linear capacity (linear stress-strain relationships) is acceptable. So no knowledge of concrete design is required! (Only the material from CVEN 305 – Mechanics of Structures)