Project Overview:

A more detailed investigation into the dynamic response of a bell tower is to be performed. The tower and bell system are now to be modeled as a two-degree-of-freedom (2DOF) system, where the degrees of freedom are:

- the *lateral* displacement at the top of the tower in the plane of the drawing.
- The *rotation* of the bell relative to the vertical line defined by the tip of the tower

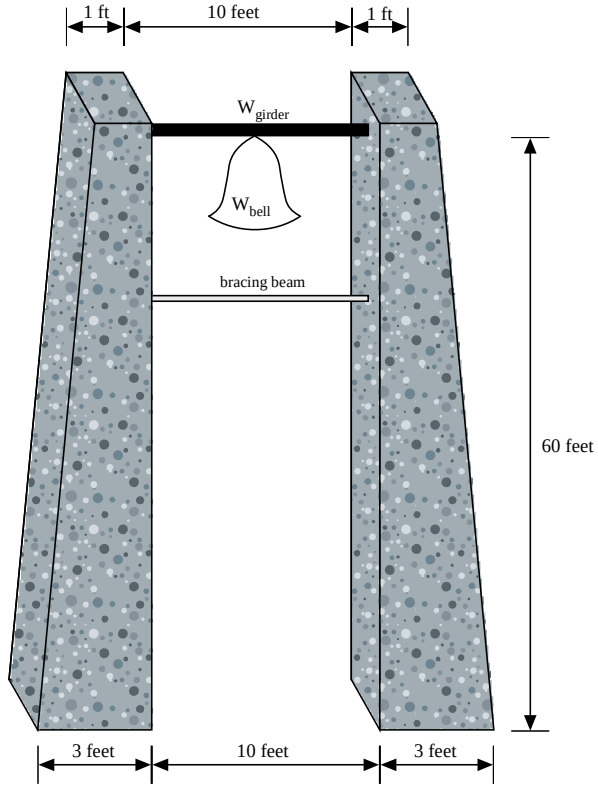


Figure 1: Bell Tower Schematic

Project Goal and Objectives:

1. Determine dynamic properties for a MDOF model of the tower and bell
   - Equivalent mass and stiffness
   - Linearized EoM for 2DOF system
   - Undamped frequencies and periods
2. Determine the dynamic response of the tower to an earthquake load
   - Given assumed damping ratio to account for energy loss and at rest initial conditions
3. Determine the combined static and dynamic forces/moments at the base of the tower.

System Description:

The bell tower to be analyzed supports one large bell with a system of tapered concrete columns. As only the vibration in a specific plane of motion is to be analyzed, only two of the columns contribute to the lateral stiffness of the system. A drawing of the tower is given to the right of this paragraph.

Bell

- The bell being supported weighs 2750 pounds and is composed of brass.
- The bell is to be modeled as a cut conical section. The diameter at the top is 2 feet and the thickness is 3 inches throughout.
- The top of the bell is to be modeled as a flat surface, and the sides flare out at 15 degrees. The height is to be determined such that the weight of the bell is as given.
- As a simplification, the entire weight of the bell is to be lumped at its centroid, so that the system is composed of particles (and not rigid bodies)

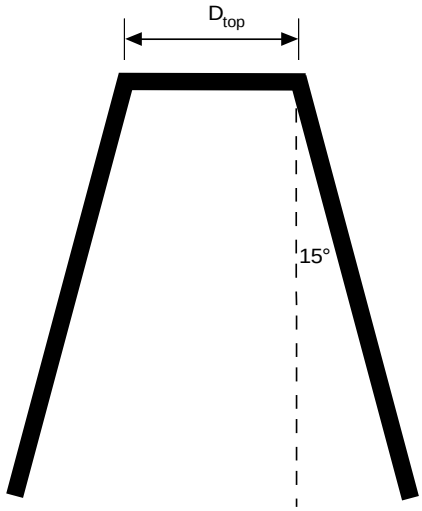


Figure 2: Bell Schematic

Supporting Girder

- It is supported by a steel girder weighing 1500 pounds.
- Girder pin-connected at each end. Will not contribute to lateral stiffness directly. Causes the towers to act as individual springs in parallel.

Bracing Beam:

- A secondary bracing beam is present for stability, but its weight and effective stiffness are neglible.

Supporting Columns:

- **Fixed support condition to their foundations**. Each tower will act as an individual cantilever beam bending about its own neutral axis. The towers then act as springs in parallel due to supporting girder.
- They are of normalweight concrete, with a density of 150 pounds per cubic foot (pcf)
- At the top, the tower's cross-section is square, with side dimensions of one foot. While the wall at the center of the tower is perfectly vertical, the outside wall flares outward, until the bottom dimension of the cross section is 3 feet by 1 foot
- The modulus of elasticity for reinforced concrete is given by:

$$E = 57,000\sqrt{f_c'} \tag{1}$$

where f_c' = 6000 psi for this structure. (You must enter the value in psi, and the resulting units of E are also psi).

Excitation Description:

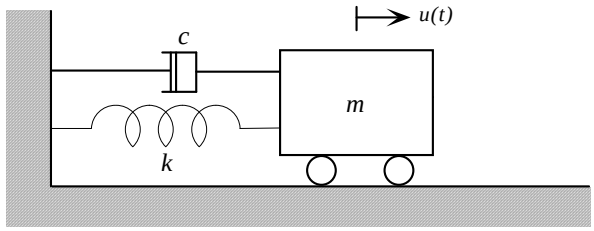
The response of the system will be performed using the data for the El Centro 1940 earthquake. A file named el_centro_data.txt is available on cenotes containing ground acceleration values in units of g measured every 0.02 seconds.

You will need to add zeros at the end of this earthquake history to allow for the system to go into free-vibration at the end of the ground motion. The typical convention is to add as many zeros as you have earthquake data points.

Detailed Tasks:

Part 3.1 – Modeling

a) Determine the equivalent mass-spring model that would represent the behavior of the system as described. In other words, for Project Part 1 the simplified physical model of the tower looked like the following figure:



For this project, what is the simplified physical model?

b) Determine the contributions to the mass from the bell. For this model, the bell is assumed to move relative to the tower. The entire mass is to be lumped at the bell's centroid, which needs to be calculated. The bell can then be modeled as a pendulum component.

c) Determine the contributions to the mass from the girder that supports the bell. For this model, the girder is assumed to move exactly with the tower and its full weight of the bell will be utilized.

d) Determine the contributions to the mass from the columns that should be lumped at the tip of the columns. This should be the exact same value as found in Project 1.

e) Determine the effective lateral stiffness of the *tapered* columns bending about their own Neutral Axis. You will again need to utilize the deflection equation of the tower due to a concentrated load applied at the tip. Find the stiffness for each tower, then combine them as springs in parallel. Recall from part 2, that concrete structures are frequently modeled using cracked section properties, you may need to evaluate both models (cracked and uncracked) of the structure.

Part 3.2 – Dynamic Equation and Properties

a) Determine the linearized equations of motion for the system using
   - the *relative* *lateral* displacement at the top of the tower in the plane of the drawing.
   - The *rotation* of the bell relative to the vertical line defined by the tip of the tower

   Be sure and place your equations in matrix form and include that EoM in the body of your memo.

b) Determine the undamped natural frequencies, the corresponding undamped periods and the mode shapes for this system.

c) Transform the EoM to Modal Space, normalizing the mode shape so the value at the rotational dof is one.

d) Some energy loss will occur in this system due to cracking of the concrete, friction at connections, etc… To account for the energy loss, a damping ratio of 5% in all modes is assumed. Determine the corresponding damping matrix in modal space.

Part 3.3 – Displacement and Force Response

a) Determine the dynamic response of your system (displacement and rotation). Take your initial conditions to be zero for relative lateral motion.

b) Develop a plot for the response. Comment/compare the plots obtained. Some things to consider: the change in amplitude of vibration, the change in frequencies, and how quickly the response dies down to zero.

c) If a rotational spring added between the bell and tower (to account for the fact that the connection is not truly frictionless), what value of rotational stiffness would result in a reduction by half of the bell's swing?

d) Utilizing the response results, actual horizontal and vertical reactions at the base of the columns due to dynamic behavior only for the system with and without the rotational spring. Show the results in a plot.

e) As the columns must also carry static loads, determine the actual horizontal and vertical reactions at the base of the columns due to the combined static and dynamic loads for the system with and without the rotational spring. Show the results in a plot.

f) Tabulate the following values for the system with and without the rotational spring:
   - Peak lateral displacement of tower tip; peak slope/rotation at tower tip
   - Peak rotation of the bell (does small angle assumption hold?)
   - Peak shear force in the tower
   - Peak moment in the tower

Part 2.4 – System evaluation and recommendation

Based on the previous results, comment on how well you believe the models capture the response of the system. Additionally, consider how well the system behaves under the loads: Does it seem sufficient? Overdesigned? Underdesigned? What changes would you recommend to the system and why? Keep in mind the following:

- Concrete can be assumed to crack in tension when tensile flexural stresses reach: $f_r = 7.5\sqrt{f_c'}$, where f_c' must be entered in psi.
- Concrete is deemed to fail when the strain in compression reaches 0.003
- Ultimate shear force capacity can be found by $2bd\sqrt{f_c'}$ -- for simplification, take:

  b = width of column (1 ft)
  h = depth of cross-section (varies from 1 ft at top to 3 ft at bottom)
  $d \approx h - 3$ inches

Note: concrete behaves nonlinearly. While a linear analysis using cracked section properties is allowed by code, the capacity is determined accounting for the nonlinear behavior and knowing the steel reinforcement. For the purposes of this project, assume that a first-pass estimate utilizing linear capacity (linear stress-strain relationships) is acceptable. So no knowledge of concrete design is required! (Only the material from CVEN 305 – Mechanics of Structures)