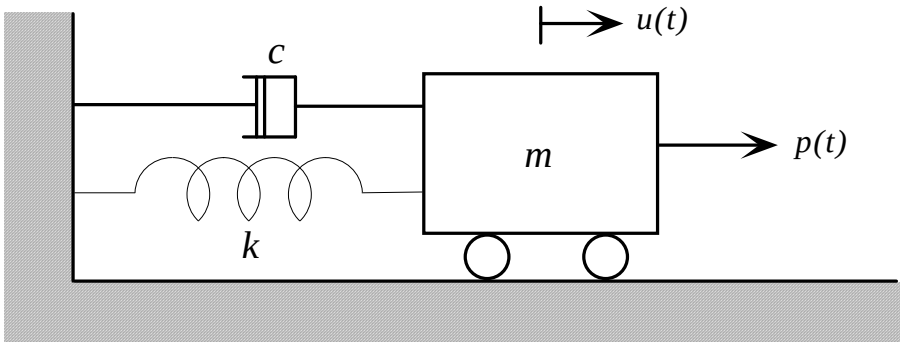
Part 1 – Modeling

The system is to be modeled, for a simplified approximation, as a heavy weight at the end of a cantilever beam. For a cantilever beam with a concentrated weight at the end, a corresponding system is that of a simple spring-mass-damper (SMD) model for horizontal motion as shown in the figure below. ***The displacement of interest is that at the bell location – or the tip of the tower – so our degree of freedom is a function of that displacement***. The first part of the project is determining effective mass and stiffness values that can be used in the simple SMD model.



a) Determine the contributions to the mass from the bell. For this model, the bell is assumed to move exactly with the tip of the tower (as opposed to being able to swing independently of the tower – which would require additional degrees of freedom). The bell is a modeled as a rigid point mass, so it will not contribute to the lateral stiffness.

b) Determine the contributions to the mass from the girder that supports the bell. For this model, the girder is assumed to move exactly with the tower, so what percentage of its full weight of the bell will experience the acceleration captured by our degree of freedom.

   The girder is taken to be pin-connected at either end – so it will not directly contribute to the lateral stiffness of the system, its impact is in the overall coupling behavior of the two towers.

c) Determine the contributions to the mass from the columns. You must determine the percentage of its full weight that will experience the acceleration captured by our degree of freedom. For example, if the full weight of the column were used, that would be equivalent to taking the entire mass of the columns and lumping it at the top. That would assume that all of the tower experiences the same acceleration – Is this a realistic assumption?

   Explore options for determining a reasonable fraction of the mass. If you find something in a book or other reference, be sure to note your source. If your team is not sure of the exact percentage, then a range of percentages will need to be explored.

d) Determine the effective lateral stiffness of the *tapered* columns.

Remember: An elastic body undergoing deformation under the action of external forces (*whether static or dynamic*) sets up internal forces within the system to resist the deformation (and so must act in opposition to the motion). These restoring forces, or spring forces, can be linear or nonlinear in nature. If the restoring forces can be assumed to be linear, then the simplest representation of an element is through the use of linear springs, where the spring stiffness constant is a property of the element material and geometric properties.

Since we are interested in the behavior at the top of the tower, that is the location where all the forces (inertial force, spring force, damper force) will be applied. You need to determine the columns resistance to a point force applied at the tip.

Hint: The effective stiffness is **not** given by $\frac{3EI}{L^3}$ in this case.