Project Background:

A bell tower is a good example of a structural system that is highly dynamic in nature. Ringing a bell can impose large dynamic forces onto the tower and connecting structures, inducing them to sway. When several bells are mounted, where each bell is rung at a different speed, the resulting motion is particularly complex.

Recently, this problem was encountered when a bell tower was added to a church cathedral in Dallas. While data from the bell manufacturers was given on the expected effective load of the bells on the tower, significant vibrations were observed when the bells were tested. At that point, a detailed dynamic investigation was required to ensure the safety of the tower and supporting church building. The pictures below are of the bell system installed in the cathedral (courtesy of: Steve Lucy, Principal, Jaster-Quintanilla, Dallas, Texas)

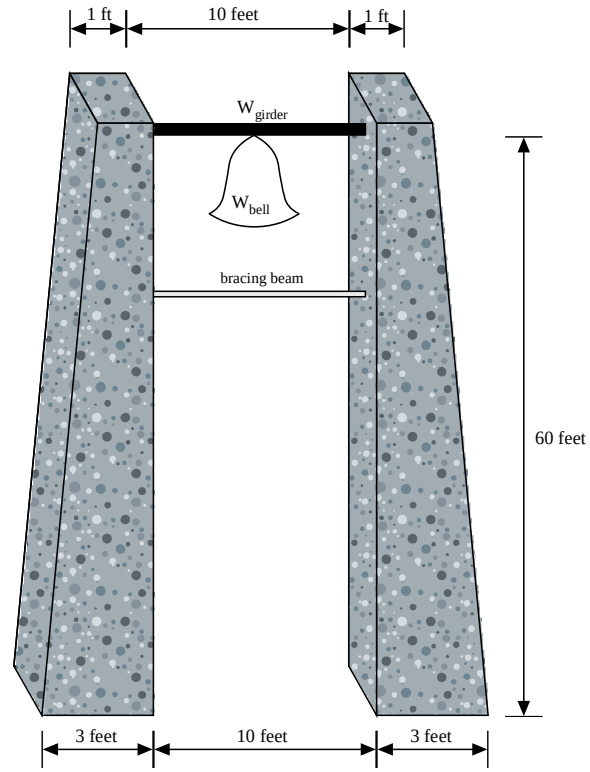
Project Overview:

An investigation into the dynamic response of a bell tower is to be performed over the course of the semester. Our bell tower will be built directly on the ground – this avoids consideration of dynamic interaction between the tower and any supporting structures. As a first, simplified approximation, the tower and bell system are modeled as a single-degree-of-freedom (SDOF) cantilever system, where the degree of freedom is the *lateral* displacement at the top of the tower in the plane of the drawing on the next page.

Project Goal and Objectives:

1. Determine dynamic properties for a SDOF model of the tower and bell
    - Equivalent mass and stiffness
    - Undamped frequency and period
2. Determine the dynamic response of the tower to a harmonic base load
    - Viscous damping model to account for energy loss and at rest initial conditions
3. Determine the peak demands:
    - Peak deformations
    - Combined static and dynamic forces/moments at the base of the tower.

System Description:



The bell tower to be analyzed supports one large bell with a system of tapered concrete columns. As only the vibration in a specific plane of motion is to be analyzed, only two of the columns contribute to the lateral stiffness of the system. A drawing of the tower is given to the right of this paragraph.

The bell being supported weighs 2750 pounds. It is supported by a steel girder weighing 1250 pounds. A secondary bracing beam is present for stability, but its weight is negligible.

The supporting columns are ***each*** assumed to be fixed-connected to their foundation, and the combination of bracing beam and steel girder are pinned connected and axially rigid. The towers are of made of normal-weight reinforced concrete, with a density of 150 pounds per cubic foot (pcf). At the top, the tower's cross-section is square, with side dimensions of one foot. While the wall at the center of the tower is perfectly vertical, the outside wall flares outward, until the bottom dimension of the cross section is 3 feet by 1 foot. The modulus of elasticity for normal-weight reinforced concrete is given by:

$$E = 57,000\sqrt{f_c'} \tag{1}$$

where f_c' = 5000 psi for this structure. (You must enter the value in psi, and the resulting units of E are also psi).

Excitation Description:

The tower is excited by (1) horizontal harmonic forces due to motors required to ring the bell, and (2) a horizontal ground motion acting simultaneously at the base of both columns. An accelerometer is typically used to record ground accelerations, and as a first order approximation it will be modeled as a harmonic function given by:

$$\ddot{x}_g(t) = \ddot{x}_0 \sin\left(\bar{\omega} t\right) \tag{2}$$

Where:

$\ddot{x}_g(t)$	=	Time-history of the ground acceleration (measured by accelerometer)
$\ddot{x}_0$	=	Peak ground acceleration
$\bar{\omega}$	=	Circular natural frequency of the ground motion (in radians/sec)