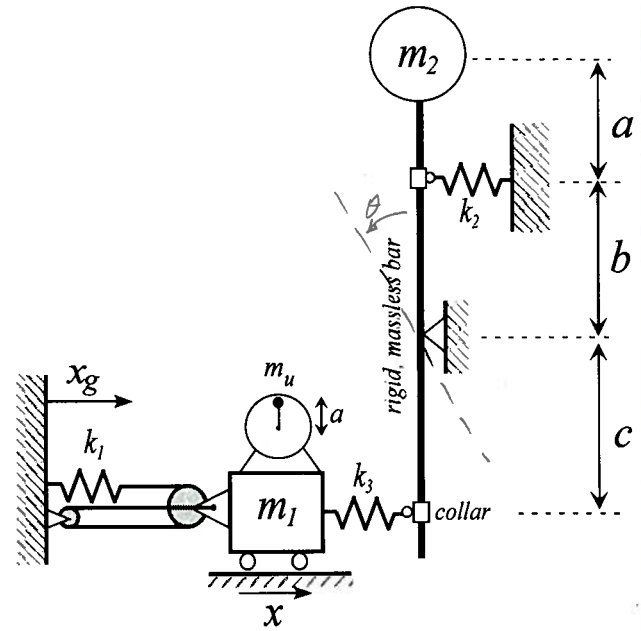
Problem 2: (25 points) Determine the matrix equation of motion for the following system *assuming small rotations* of the bar. One degree of freedom is already labeled – you need to decide if any others exist and what they are. The system is excited by the following:

- A ground motion at the left support described by $x_g(t)$
- A motor with a rotating unbalanced mass of magnitude m_u located at an eccentricity of a. The motor rotates clockwise at a frequency of $\bar{\omega}$.



VARIABLES

x = Total horizontal motion of m_1 from static equilib.

θ = Total rotation of bar supporting m_2 about pivot in CCW direction

Δ_1 = deformation spring 1

Δ_2 = deformation spring 2

Δ_3 = deformation spring 3

x_m = horizontal motion m_u RELATIVE position of m_1

DOFs: x & θ

Kinematics

$$\Delta_1 = 3(x - x_g)$$

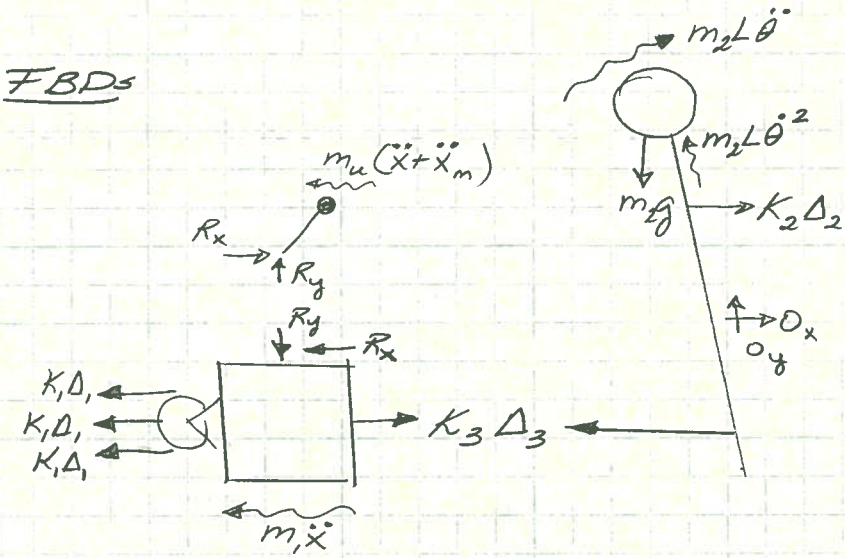
$$\Delta_3 \cong c\theta - x \qquad \text{assuming } c\theta > x \therefore \text{ spring in tens.}$$

$$\Delta_2 \cong b\theta$$

$$L = a + b \quad \text{(distance from pivot to } m_2\text{)}$$

$$x_m = a\cos(\bar{\omega}t) \qquad \therefore \ \ddot{x}_m = -a\bar{\omega}^2\cos(\bar{\omega}t)$$

FBDs



Equilibrium

① $m_1\ddot{x} + 3K_1\Delta_1 + R_x - K_3\Delta_3 = 0$

② $m_u(\ddot{x} + \ddot{x}_m) - R_x = 0$

③ $m_2L^2\ddot{\theta} + K_2\Delta_2 b + K_3\Delta_3 c - m_2gL\theta = 0$

Combining ① and ② to eliminate R_x

$$m_1\ddot{x} + m_u(\ddot{x} + \ddot{x}_m) + 3K_1\Delta_1 - K_3\Delta_3 = 0$$

Substitute kinematics

$$(m_1 + m_u)\ddot{x} + 3K_1(3(x - x_g)) - K_3(c\theta - x) = -m_u\ddot{x}_m$$

④ $(m_1 + m_u)\ddot{x} + (9K_1 + K_3)x - (K_3c)\theta = 9K_1x_g + m_ua\bar{\omega}^2\cos(\bar{\omega}t)$

Substitute Kinematics into (3)

$$(m_2L^2)\ddot{\theta} + K_2b^2\theta + K_3c(c\theta - x) - (m_2gL)\theta = 0$$

(5) $$(m_2L^2)\ddot{\theta} + (K_2b^2 + K_3c^2 - m_2gL)\theta - (K_3c)x = 0$$

Writing equations (4) & (5) in matrix

$$\begin{bmatrix} (m_1+m_u) & 0 \\ 0 & (m_2L^2) \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (9K_1+K_3) & -K_3c \\ -K_3c & (K_2b^2+K_3c^2-m_2gL) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 9K_1 \\ 0 \end{Bmatrix} x_g + \begin{Bmatrix} m_u a\bar{\omega}^2 \\ 0 \end{Bmatrix} \cos(\bar{\omega}t)$$