Problem: Derive the ***fully nonlinear equation of motion*** for the system shown below. You may NOT assume small deformations for any portion of this problem. You may assume that

1. Springs 2 and 3 are pre-stressed so the cables never go slack (i.e. – they are initially in tension).
2. When at equilibrium, spring 1 is undeformed and oriented perfectly vertical.
3. There is no friction in any part of the system and the pulleys are massless.

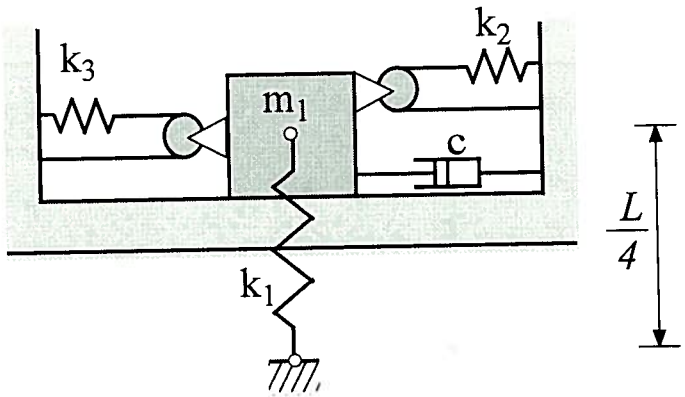
Be sure to:

a) Clearly and completely define your degree of freedom.

b) Write all kinematic constraint relationships immediately after defining your dof. Specifically, clearly write a relationship describing the stretch in each spring as a function of your dof.

c) Draw all relevant Free-Body Diagrams, indicating if using Total Forces or Incremental Dynamic Forces. If you use strict application of Newton's 2nd Law, you must also draw the Mass-Acceleration Diagram. If you use D'Alembert's Principle, show the inertia force(s) using a wavy or dashed arrow so as to clearly indicate that this force is a fictitious force.

Apply equilibrium principles and get the nonlinear equation of motion. BOX your final answer.



DOF

x = horizontal motion of m_1 from static equilibrium

Other deformations

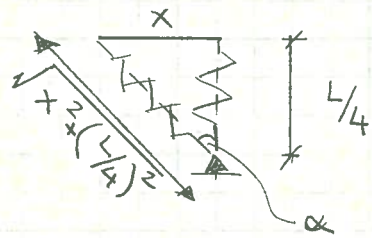
x_{s1} = motion @ end of spring 1 ALONG the direction of the spring (NOT ALWAYS VERTICAL) from static equilibrium position

x_{s2} = horizontal motion @ end of spring 2 from static equilibrium

x_{s3} = horizontal motion @ end of spring 3 from static equilibrium

Kinematic Constraints

$$x_{s1} = \sqrt{x^2 + (L/4)^2} - (L/4)$$



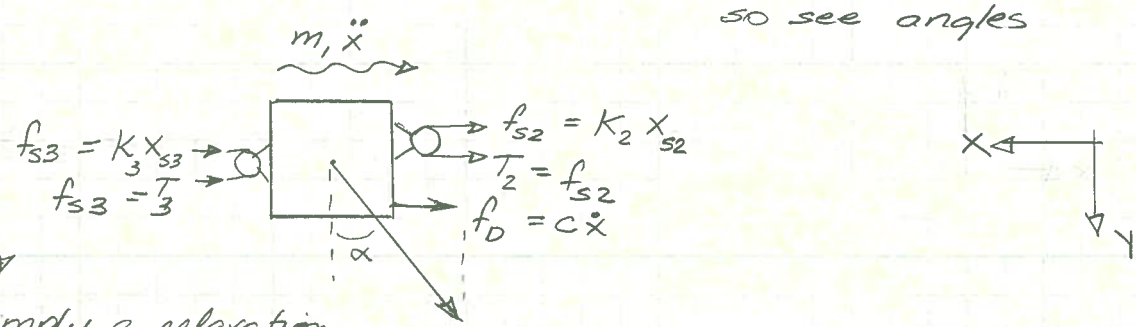
$$x_{s2} = 2x$$

$$x_{s3} = 2x$$

$$\tan\alpha = \frac{x}{(L/4)} = \frac{4x}{L}$$

$$\sin\alpha = \frac{x}{\sqrt{x^2+(L/4)^2}}$$

FBD - Dynamic Forces Only. Remember - draw in DEFORMED position so see angles



imply a relaxation in total cable force

$$f_{s1} = K_1\left[\sqrt{x^2 + (L/4)^2} - (L/4)\right]$$

$$\sum F_x = 0 \qquad m_1\ddot{x} + f_{s2} + T_2 + f_{s1}\sin\alpha + f_{s3} + T_3 + f_D = 0$$

$$m_1\ddot{x} + 2f_{s2} + f_{s1}\left(\frac{x}{\sqrt{x^2+(L/4)^2}}\right) + 2f_{s3} + f_D = 0$$

$$m_1\ddot{x} + 2K_2 x_{s2} + 2K_3 x_{s3} + \frac{K_1 x}{\sqrt{x^2+(L/4)^2}}\left[\sqrt{x^2+(L/4)^2} - (L/4)\right] + c\dot{x} = 0$$

$$\boxed{m_1\ddot{x} + c\dot{x} + (4K_2 + 4K_3)x + \frac{K_1 x}{\sqrt{x^2+(L/4)^2}}\left[\sqrt{x^2+(L/4)^2} - (L/4)\right] = 0}$$