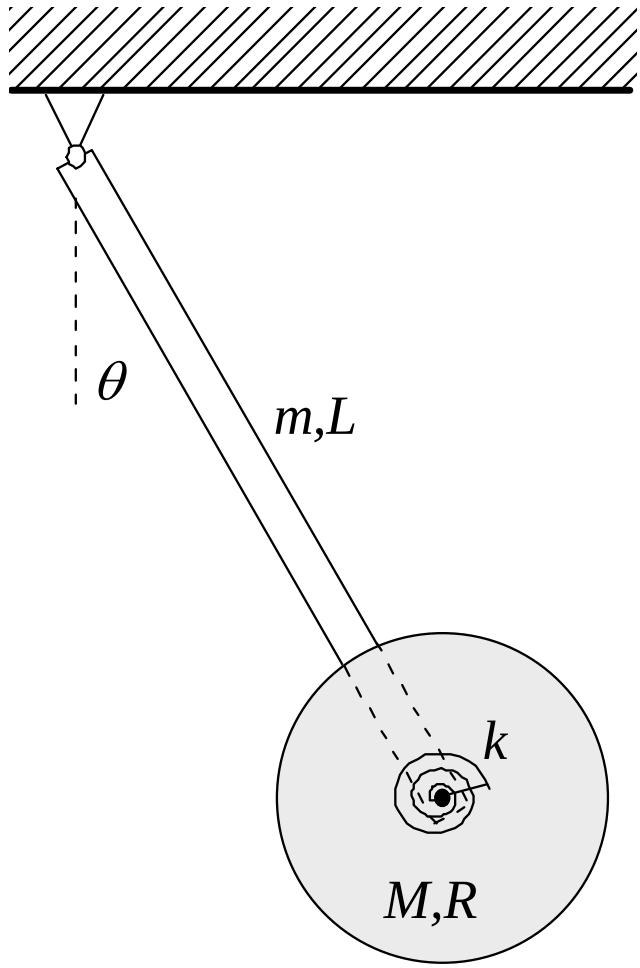
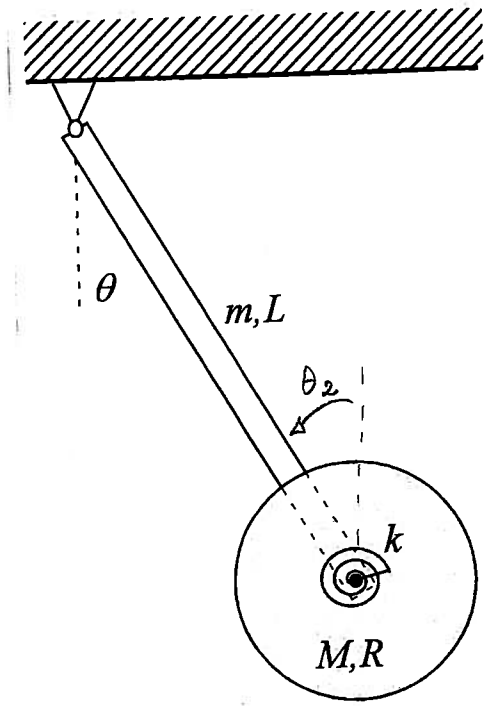
Problem 5: The compound pendulum system shown below is composed of a rectangular rigid bar of mass *m* and length *L* and a disk at its end of mass *M* and radius *R*. The disk is attached to the bar at its center so as to allow the disk to spin relative to the bar (translations at the connection are restrained). Derive the equations of motion for this system for small motions and place them in matrix form.



Prob 3//

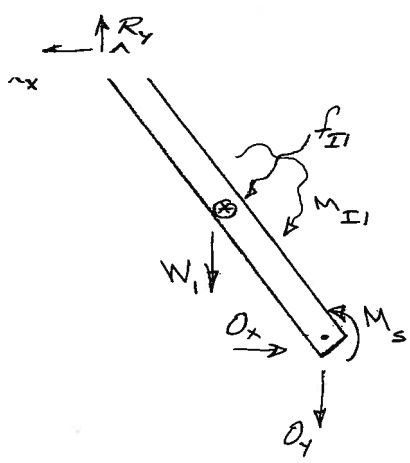


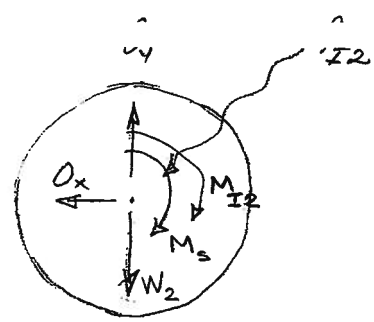
Dofs:

θ_1 = CCW rotation from vertical of rod (TOTAL)

θ_2 = CCW rotation from vertical of mass (TOTAL rotation, NOT total motion)

Assume: $\theta_2 > \theta_1$





$$f_{I1} = m\left(\frac{L}{2}\ddot{\theta}_1\right)$$

$$M_{I1} = \left(\frac{mL^2}{12}\right)\ddot{\theta}_1$$

$$W_1 = mg$$

$$M_s = k(\theta_2 - \theta_1)$$

$$f_{I2} = M(L\ddot{\theta}_1)$$

$$M_{I2} = \left(\frac{MR^2}{2}\right)\ddot{\theta}_2$$ total spin of mass

$$W_2 = Mg$$

From FBD (B)

$$M_{I2} + M_s = 0 \qquad (1)$$

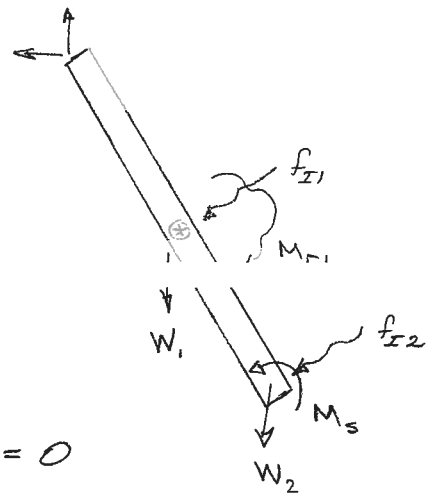
$$-O_x + f_{I,2}\cos\theta_1 = 0 \qquad (2)$$

$$O_y - W_2 - f_{I,2}\sin\theta_1 = 0 \qquad (3)$$

Subs into (1) and:

$$\left(\frac{MR^2}{2}\right)\ddot{\theta}_2 + k(\theta_2 - \theta_1) = 0 \qquad (4)$$

From FBD (A')



Take moments about hinge

$$(f_{I1})\left(\frac{L}{2}\right) + M_{I1} + W_1\left(\frac{L}{2}\sin\theta_1\right)$$
$$+ f_{I2}(L) + W_2(L\sin\theta_1) - M_s = 0$$

$$\left(\frac{mL^2}{4}\right)\ddot{\theta}_1 + \left(\frac{mL^2}{12}\right)\ddot{\theta}_1 + mg\left(\frac{L}{2}\right)\theta_1 + ML^2\ddot{\theta}_1 + MgL\theta_1 - k(\theta_2 - \theta_1) = 0$$

$$\left(\frac{mL^2}{3} + ML^2\right)\ddot{\theta}_1 + \left(\frac{mgL}{2} + MgL\right)\theta_1 - k(\theta_2 - \theta_1) = 0$$

$$\left(M + \frac{m}{3}\right)L\ddot{\theta}_1 + \left(M + \frac{m}{2}\right)g\,\theta_1 + \frac{k}{L}\theta_1 - \frac{k}{L}\theta_2 = 0$$

$$\left[\left(M + \frac{m}{3}\right)L\right]\ddot{\theta}_1 + \left[\left(M + \frac{m}{2}\right)g + \frac{k}{L}\right]\theta_1 - \left[\frac{k}{L}\right]\theta_2 = 0$$

In matrix form:

$$\begin{bmatrix} \left(M+\frac{m}{3}\right)L & 0 \\ 0 & \frac{MR^2}{2} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} \left(M+\frac{m}{2}\right)g + \frac{k}{L} & -\frac{k}{L} \\ -k & k \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Note that stiffness mx. can be made symmetric by mult. top eqn by L.

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