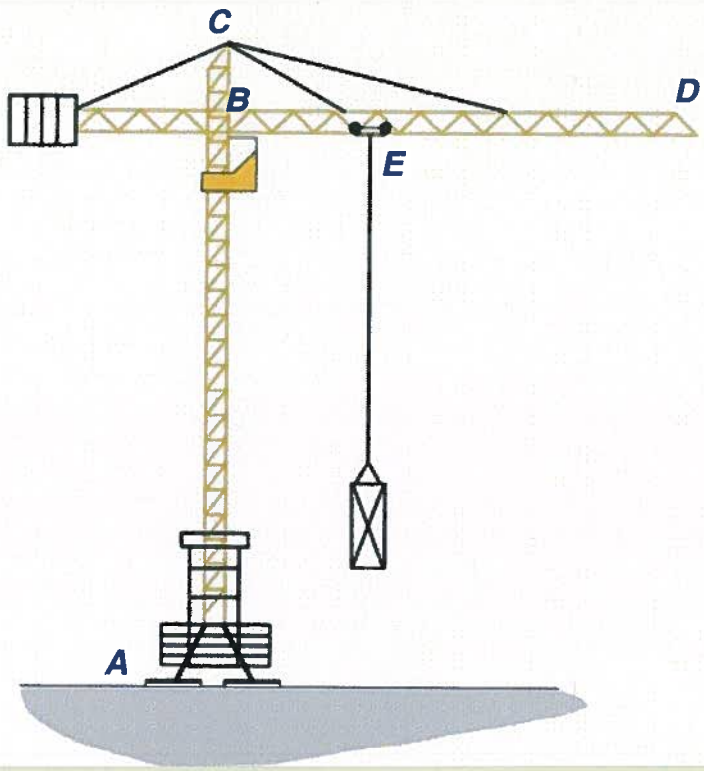
Problem 1: A crane, shown below, is composed of a fixed supporting tower (ABC) and a rotating arm (BD). The load is supported by a rolling cart as indicated by E. The location of the cart is initially 30 feet away from the tower support (position B). To move the load, the operator rotates the arm at a rotational rate of $\dot{\theta}$, which increases linearly from zero to a maximum value of 1 rpm. At the same time, the operator moves the cart inward towards B at a rate along the supporting arm of $1+0.5t$ inches per second to a limiting value of 15 inches per second in magnitude. The limiting values in both radial and rotational directions are reached at the same time.

a) How long does it take for cart to be within 1 foot of the supporting tower (position B)?
b) What is the magnitude of velocity of the cart at 1 foot away from B?
c) What is the magnitude of acceleration of the cart at 1 foot away from B?



Solution

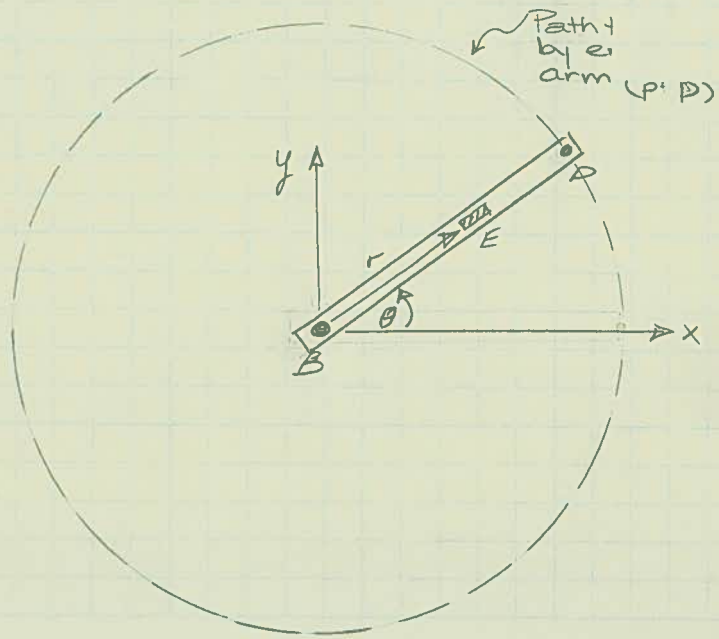
r = radial distance from B to E

θ = CCW angle of arm from horizontal position

@ $t = 0$

$r(t=0) = r_0 = 30$ ft

$\theta(t=0) = \theta_0$



Rotational Properties

$$\omega_{max} = \left(\frac{1\ rot}{min}\right)\left(\frac{2\pi\ rad}{1\ rot}\right)\left(\frac{1\ min}{60\ sec}\right) = 0.1047\ rad/s$$

$$\dot{\theta}(t) = \begin{cases} 0.1047\,(t/t_1)\ rad/s & 0 < t < t_1 \\ 0.1047\ rad/s & t_1 < t \end{cases}$$

$$\ddot{\theta}(t) = \begin{cases} 0.1047/t_1\ rad/s^2 & 0 < t < t_1 \\ 0 & t_1 < t \end{cases}$$

Radial Properties

Neg since 'inward' direction!

$$\dot{r}(t) = \begin{cases} -(1+0.5t)\ in/s & 0 < t < t_1 \\ -15\ in/s & t_1 < t \end{cases}$$

$$\ddot{r}(t) = \begin{cases} -0.5\ in/s^2 & 0 < t < t_1 \\ 0\ in/s^2 & t_1 < t \end{cases}$$

Determine time peak occurs

$$1 + 0.5t = 15$$
$$0.5t = 14$$
$$t = 28 \text{ seconds} \quad \therefore \quad t_1 = 28\ sec$$

So equations:

$$\dot{\theta}(t) = \begin{cases} 0.0037393\,t\ rad/s & 0 < t < 28\ sec \\ 0.1047\ rad/s & 28\ sec < t \end{cases}$$

$$\ddot{\theta}(t) = \begin{cases} 0.0037393\ rad/s^2 & 0 < t < 28\ sec \\ 0 & 28\ sec < t \end{cases}$$

$$\dot{r}(t) = \begin{cases} -1 - 0.5t\ in/s & 0 < t < 28\ sec \\ -15\ in/s & 28\ sec < t \end{cases}$$

$$\ddot{r}(t) = \begin{cases} -0.5\ in/s^2 & 0 < t < 28\ sec \\ 0 & 28\ sec < t \end{cases}$$

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a) $\frac{dr}{dt} = \dot{r}$

$$dr = \dot{r}\,dt$$

$$\int_{r_0}^{r} dr = \int_0^t \dot{r}\,dt$$

$$r - r_0 = \int_0^t (-1 - 0.5t)\,dt = -t - 0.25t^2$$

$$r = r_0 - t - 0.25t^2$$

$$(12\text{ in}) = (30\text{ ft} * 12\text{ in/ft}) - t - 0.25t^2$$

$$0.25t^2 + t - 348 = 0$$

$$t = \frac{-1 \pm \sqrt{(1)^2 - 4(0.25)(-348)}}{2(0.25)}$$

$$t = (-1 \pm 18.682)/0.5 = 35.36\text{ sec or } \cancel{-39.36\text{ sec}}$$

Only positive root physically realistic, so

$$t = 35.36\text{ sec} > 28\text{ sec}$$

But eqn used for $\dot{r}$ only $t \le 28$ sec. So; need to find how far traveled in 28 sec & use other eqn for $\dot{r}$ to determine time for remaining portion

$$r = (30' * 12\text{ in/ft}) - (28\text{ sec}) - 0.25(28\text{ s})^2$$
$$= 136\text{ in} = 11.333\text{ ft}$$

for $t > 28$ sec

$$r - r_0 = \int_{28}^{t} (-15)\,dt = -15t\Big|_{28}^{t} = -15(t - 28)$$

$$(12'') = (136\text{ in}) - 15(t - 28)$$

$$348 = 15(t - 28)$$

$$\underline{\underline{t = 36.27\text{ sec}}} \qquad > 28\text{ sec} \checkmark \text{ ok}$$

b) Magnitude of Velocity (speed)

$$\{v\} = \dot{r}\{e_R\} + r\dot{\theta}\{e_\theta\}$$

@ $t = 36.27$ sec:

$r = 12$ in
$\dot{r} = -15$ in/s
$\dot{\theta} = 0.1047$ rad/s

$$\{v\} = (-15)\{e_R\} + [(12)(0.1047)]\{e_\theta\} \text{ in/s}$$

$$= (-15)\{e_R\} + (1.2566)\{e_\theta\} \text{ in/s}$$

$$\|\{v\}\| = \sqrt{(-15)^2 + (1.2566)^2} = \underline{\underline{15.05 \text{ in/s}}}$$

c) Magnitude of Acceleration

$$\{a\} = (\ddot{r} - r\dot{\theta}^2)\{e_r\} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\{e_\theta\}$$

@ $t = 36.27$ sec

$\ddot{r} = 0$
$\ddot{\theta} = 0$

$$\{a\} = [-(12)(0.1047)^2]\{e_R\} + [2(-15)(0.1047)]\{e_\theta\}$$

$$= (-0.1315)\{e_R\} + (3.141)\{e_\theta\}$$

$$\|\{a\}\| = \sqrt{(-0.1315)^2 + (3.141)^2} = \underline{\underline{3.144 \text{ in/s}^2}}$$

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