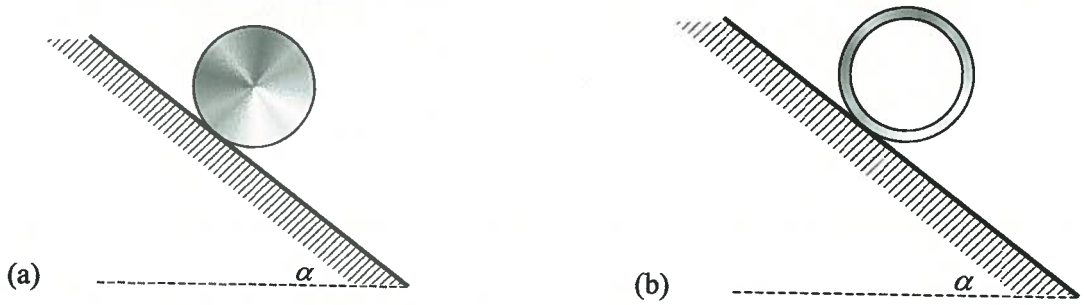
Problem 3: Two similar rigid body systems are to be analyzed: (a) a solid disk and (b) a thin circular shell. The two systems are characterized by a total mass M that is uniformly distributed, and by a cross-sectional radius R. The elements roll without slip down an inclined surface of angle α with the horizontal as shown.



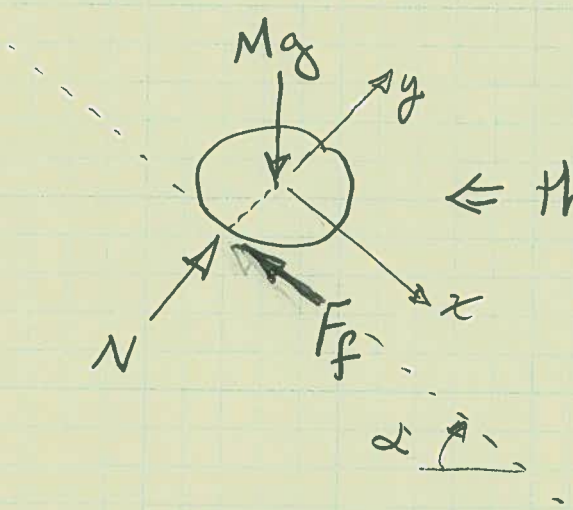
For both systems, determine:

(i) The equation for Linear Acceleration of the center of mass (C.M)
(ii) The equation for rotational acceleration about the center of mass
(iii) The value of the friction needed so that slip does not occur

Recall $I = \frac{MR^2}{2}$ for SOLID DISK and MR^2 for THIN SHELL

Newton Says: $\Sigma F = m a_{CM}$

and $\Sigma M = I \ddot{\theta}_{CM}$



⇐ this FBD can represent both

$\Sigma F_x = m a_{CMx}$

$$\boxed{Mg\sin\alpha - F_f = M a_{CMx}}$$ EQN 1

$\ddot{x}$

$\circlearrowleft \Sigma M_{CM} = I \ddot{\theta}_{CM}$

$F_f \cdot R = I \ddot{\theta}_{CM}$

so far thing every works for both

$\underline{\underline{i}}$

SOLID DISK

$$F_f \cdot R = \frac{MR^2}{2}\ddot{\theta}_{CM}$$

THIN SHELL

$$F_f \cdot R = MR^2\ddot{\theta}_{CM}$$

substituting into EQN 1

$$Mg\sin\alpha - \frac{MR}{2}\ddot{\theta}_{CM} = M\ddot{x} \quad \& \quad Mg\sin\alpha - MR\ddot{\theta}_{CM} = M\ddot{x}$$

Now we use Roll without slip $\Rightarrow$ $\theta R = x$

& since R = constant $\Rightarrow$ $\dot{\theta}R = \dot{x}$

$\ddot{\theta}R = \ddot{x}$

SOLID DISK

$$Mg\sin\alpha = \frac{3}{2}M\ddot{x}$$

$$\frac{2}{3}g\sin\alpha = \ddot{x}_{CM}$$

THIN SHELL

$$Mg\sin\alpha = 2M\ddot{x}$$

$$\frac{1}{2}g\sin\alpha = \ddot{x}_{CM}$$

&

Note: it is easier to turn the SOLID DISK (I is $\frac{1}{2}$ as big)

$\underline{\underline{ii}}$

$$\ddot{\theta}_{CM} = \frac{\ddot{x}}{R} \quad \therefore$$

SOLID DISK

$$\ddot{\theta}_{CM} = \frac{2g}{3R}\sin\alpha$$

THIN SHELL

$$\ddot{\theta}_{CM} = \frac{g}{2R}\sin\alpha$$

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Note: $F_f \neq \mu N$ $\quad (\mu N = \text{maximum } F_f)$

F_f can be found from EQNS on top of p 2/3

SOLID DISK

$$F_f = \frac{MR}{2}\ddot{\theta}$$

THIN SHELL

$$F_f = MR\ddot{\theta}$$

from bottom of p 2/3

$$F_f = \frac{MR}{2}\left(\frac{2g}{3R}\right)\sin\alpha$$

$$F_f = MR\frac{g}{2R}\sin\alpha$$

$$\boxed{F_f = \frac{Mg}{3}\sin\alpha}$$

$$\boxed{F_f = \frac{Mg}{2}\sin\alpha}$$

if this friction is not available, THEN slip will happen.