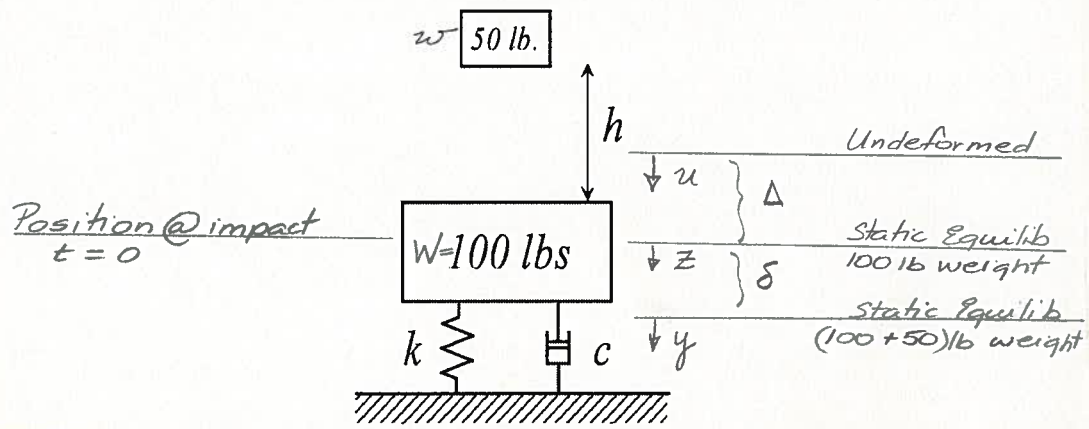
Problem 2: A 50 pound block drops a distance, *h* of 18 inches onto a 100 pound block that is originally at rest. The 50 pound block ***sticks*** to the 100 pound block on impact, and the resulting system has an undamped period of 0.5 seconds and is 5% critically damped. Then:

a) Determine the displacement response equation of the combined masses from the position of impact. (Note: you may use any DOF choice in the process). Plot the response showing at least 5 cycles of oscillation. Calculate the location of the system 1.5 seconds after impact from the position of impact.

b) Utilize the software package Tinker to verify your results. Include a screen-shot with appropriate plot(s) from the software showing. What parts of the problem can Tinker accurately simulate? If there are differences, what do you think is the cause? You may want to export the data from the Tinker plots to better compare with your analytical result.



Possible choices for degree of freedom

u = vertical motion of combined masses from undeformed spring position

z = vertical motion of combined masses from static equilib position of 100 lb weight

y = vertical motion of combined masses from static equilib position of the combined masses (100 + 50 lb weights)

Depending on your dof choice, you will have:

1) a different EOM

2) a different set of initial conditions.

Relationship between coordinates:

$$u = z + \Delta = y + \Delta + \delta \qquad \dot{u} = \dot{z} = \dot{y}$$
$$z = y + \delta$$

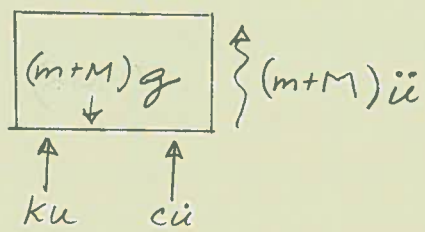
velocity eqns same

a) Determine displacement response from impact position

<u>OPTION 1</u>: using u as dof

<u>DOF = u</u> (defined page 1)

<u>FBD</u> - total forces



$$m = \frac{50\,lb}{386.4\,in/s^2} = 0.1294\ \frac{lb\text{-}s^2}{in}$$

$$M = \frac{100\,lb}{386.4\,in/s^2} = 0.2588\ \frac{lb\text{-}s}{in}$$

<u>Apply equilibrium</u>

$$(m+M)\ddot{u} + c\dot{u} + ku = \underbrace{(m+M)g}_{F_0} \quad \leftarrow EoM$$

<u>Response</u>: Forced vibration to const load

$$u(t) = e^{-\zeta\omega t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right) + \frac{\overbrace{(m+M)g}^{F_0}}{K}$$

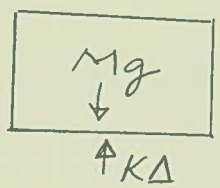
where: $A = u_0 - \frac{F_0}{K}$ $\quad B = \frac{\dot{u}_0 + \zeta\omega u_0}{\omega_d} - \frac{\zeta\omega F_0}{\omega_d K}$

<u>Need initial conditions for this dof</u>

$u_0 = u(t=0) = +\Delta$ ← find through statics

$\dot{u}_0 = \dot{u}(t=0)$ ← from impact w/ falling block

• STATICS



$$Mg = K\Delta$$

$$\Delta = \frac{Mg}{K} = \frac{W}{K}$$

Need K → related to undamped nat period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m^*}{K^*}}$$

$$T^2 = \frac{4\pi^2 m^*}{K^*}$$

$$(K = K^*) = \frac{4\pi^2 m^*}{T^2}$$

$$K = 4\pi^2(0.1294 + 0.2588)/(0.5s)^2$$

$$K = 61.302 \text{ lb/in}$$

So: $\Delta = \frac{100 \text{ lbs}}{61.3 \text{ lb/in}} = 1.6313$ inches

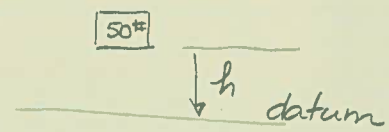
$u_0 = 1.6313$ in

- Initial velocity -

→ velocity of 50 lb block just before impact

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2 + 0$$



$$v = \sqrt{2gh}$$

$$v = \sqrt{2(386.4)(18)} = 117.942 \text{ in/s}$$

→ velocity just after impact

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v_0$$

$$(0.1294)(117.942) = (0.1294 + 0.2588)v_0$$

$$39.3141 = v_0$$

So $\dot{u}_0 = +39.3141$ in/sec

Frequencies:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5s} = 12.566 \text{ rad/s}$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 12.566\sqrt{1-(0.05)^2}$$
$$= 12.55 \text{ rad/s}$$

$$A = u_0 - F_0/k = 1.6313 - 150/61.3 = -0.816 \text{ in}$$

$$B = \frac{\dot{u}_0 + \zeta\omega u_0}{\omega_d} - \frac{\zeta\omega F_0}{\omega_d k}$$
$$= 3.0916 \text{ in}$$

So response from undeformed position

$$u(t) = e^{-0.6283t}\left(-0.816\cos(12.55t) + 3.092\sin(12.55t)\right) + 2.4469$$

Need response from location of impact

$$z(t) = u(t) - \Delta$$
$$z(t) = e^{-0.6283t}\left(-0.816\cos(12.55t) + 3.092\sin(12.55t)\right) + 2.4469 - 1.631$$

$$\boxed{z(t) = e^{-0.6283t}\left(-0.816\cos(12.55t) + 3.092\sin(12.55t)\right) + 0.8156 \text{ inches}}$$

@ $t = 1.5$ seconds

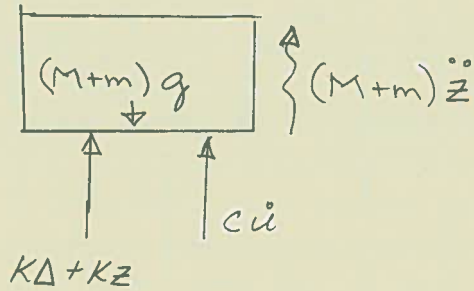
$z(t \doteq 1.5 \text{ sec}) = +0.4695$ inches

So @ 0.47 inches below impact position

OPTION 2 - using z as dof

DOF = z (defined page 1)

FBD - total forces



$m = 0.1294\ lb\text{-}s^2/in$

$M = 0.2588\ lb\text{-}s^2/in$

$K = 61.302\ lb/in$ (pg 3)

Apply equilibrium

$$(M+m)\ddot{z} + c\dot{z} + Kz + K\Delta - (M+m)g = 0$$

$$(M+m)\ddot{z} + c\dot{z} + Kz + \underbrace{\cancel{K\Delta} - \cancel{Mg}}_{\text{cancel due to statics - see pg 2}} - mg = 0$$

$$(M+m)\ddot{z} + c\dot{z} + Kz = \underbrace{+mg}_{f_0} \quad \Leftarrow \text{EOM}$$

Response: Forced vibration to const. load

$$z(t) = e^{-\zeta\omega t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right) + \frac{f_0}{K}$$

where: $A = z_0 - \frac{f_0}{K}$ and $B = \frac{\dot{z}_0 + \zeta\omega z_0}{\omega_d} - \frac{\zeta\omega f_0}{\omega_d K}$

Need initial conditions for THIS dof.

$z_0 = z(t=0) = 0$ ← @ $t=0$ exactly @ this pt.

$\dot{z}_0 = \dot{z}(t=0)$ ← from impact

Initial Velocity (same as on pg 3)

→ velocity of 50 lb block just before impact

$$T_1 + V_1 = T_2 + V_2$$

$$v = \sqrt{2gh} = \sqrt{2(386.4)(18)} = 117.942 \text{ in/s}$$

→ velocity after impact

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_0$$

$$(0.1294)(117.42) = (0.1294 + 0.2588) v_0$$

$$v_0 = 39.3141 \text{ in/sec}$$

So: $\dot{z}_0 = +39.3141$ in/s ← Same velocity see relationship eqns @ bottom page 1.

Frequencies: $\omega = 12.566$ rad/sec, $\omega_d = 12.55$ rad/sec } pg 4

$$A = z_0 - \frac{f_0}{K} = 0 - \frac{mg}{K} = -\frac{50 \text{ lbs}}{61.302} = -0.816 \text{ in}$$

$$B = \frac{\dot{z}_0 + \zeta\omega \cancel{z_0}^{\,0}}{\omega_d} - \frac{\zeta\omega f_0}{\omega_d K} = 3.0916 \text{ in}$$

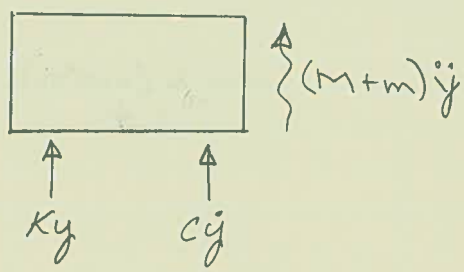
$$\frac{f_0}{K} = \frac{50 \text{ lbs}}{61.302} = +0.816 \text{ in}$$

$$\boxed{z(t) = e^{-0.628t}\left(-0.816\cos(12.55t) + 3.092\sin(12.55t)\right) + 0.816 \text{ inches}}$$

OPTION 3 - using y as your dof

DOF $= y$ (defined pg 1)

FBD - Dynamic Only Forces



$m = 0.1294 \frac{lb\text{-}s^2}{in}$

$M = 0.2588 \frac{lb\text{-}s^2}{in}$

$k = 61.302 \; lb/in$ (pg 3)

Apply equilibrium

$$(M+m)\ddot{y} + c\dot{y} + ky = 0 \quad \Leftarrow \text{EOM}$$

Response: Free vibration due to initial cond.

$$y(t) = e^{-\zeta\omega t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

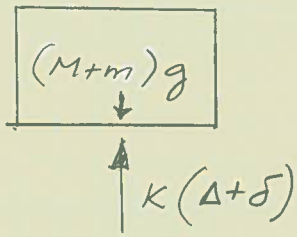
where: $A = y_0$ and $B = \frac{\dot{y}_0 + \zeta\omega y_0}{\omega_d}$

Need initial conditions for this dof

$y_0 = y(t=0) = -\delta$ ← from statics

$\dot{y}_0 = \dot{y}(t=0)$ ← from impact of falling block

- STATICS



$$\Sigma F = 0$$

$$(M+m)g = k(\Delta+\delta)$$

$$\cancel{Mg} + mg = \cancel{k\Delta} + k\delta$$

See FBD on pg 2

$$mg = k\delta$$

$$\frac{w}{k} = \frac{mg}{k} = \delta$$

$$\delta = \frac{50\,lbs}{61.302\,lb/in} = 0.8156 \text{ inches}$$

$$y_0 = -\delta = -0.8156 \text{ inches}$$

- Initial velocity

$$T_1 + V_1 = T_2 + V_2$$

$$v = \sqrt{2gh}$$

$$v = 117.942 \text{ in/s}$$

See details pg 3

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_0$$

$$v_0 = 39.3141 \text{ in/s}$$

See details pg 3

So $\dot{y}_0 = +39.3141$ in/s

Frequencies

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.566 \text{ rad/s}$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 12.566\sqrt{1-(0.05)^2}$$

$$= 12.55 \text{ rad/s}$$

Constants:

$$A = y_0 = -0.8156 \text{ inches}$$

$$B = \frac{\dot{y}_0 + \zeta\omega y_0}{\omega_d} = \frac{39.3141 + (0.05)(12.566)(-0.8156)}{12.55}$$

$$= 3.0916 \text{ inches}$$

So response from static equilibrium of both masses:

$$y(t) = e^{-0.6283t}\left(-0.8156\cos(12.55t) + 3.0916\sin(12.55t)\right)$$

Need response from position of impact

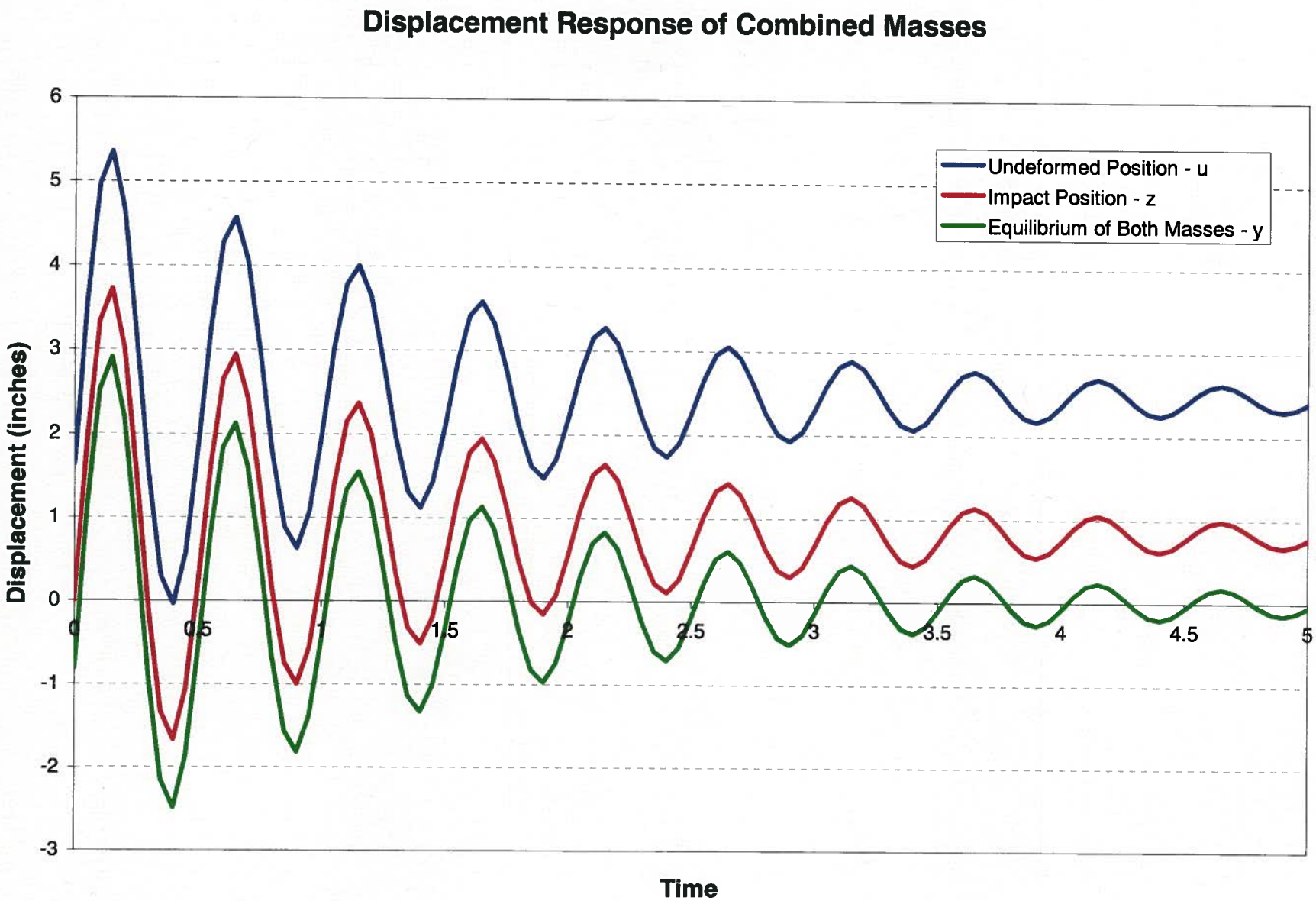
$$z(t) = y + \delta$$

$$\boxed{z(t) = e^{-0.6283t}\left(-0.8156\cos(12.55t) + 3.0916\sin(12.55t)\right) + 0.8156 \text{ inches}}$$

@ $t = 1.5$ seconds

$$z(t=1.5) = +0.4695 \text{ inches}$$

<u>So @ 0.47 inches below impact position</u>


Displacement Response of Combined Masses
Undeformed Position - u
Impact Position - z
Equilibrium of Both Masses - y
Displacement (inches)
Time
6
5
4
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5