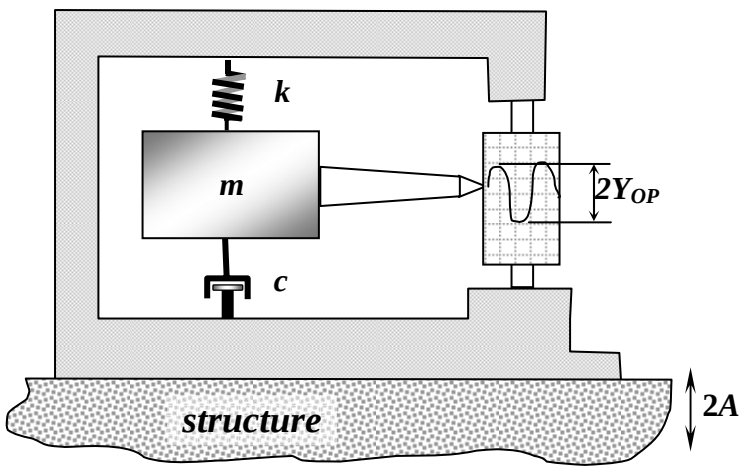
Problem 2: The system shown is a configuration for a seismic motion measurement system. The system is mounted on a structure that has a vertical harmonic vibration of 15 Hz and double amplitude, $2A$, of 2 mm. The sensing element has a mass m = 2.0 kg, and the spring has a stiffness k = 1.75 kN/m . The motion of the mass <u>*relative*</u> to the instrument base is recorded on a revolving drum and shows double amplitude, $2Y_{OP}$, of 2.2 mm during the steady-state condition.

Task:

(a) Derive the EoM for the system. Use for your degree of freedom the deformation variable whose response corresponds to the motion plotted by the rotating drum.

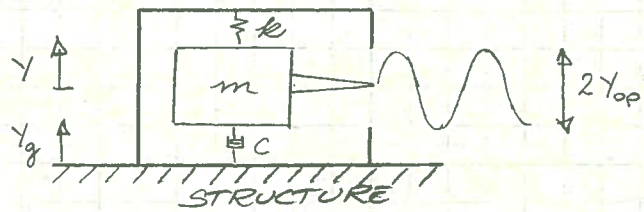
(b) Calculate the viscous damping constant c

.



<u>*Hint*</u>: you must decide if the response plotted by the rotating drum corresponds to total/absolute motion of the system or the relative motion of the system?

Problem 3.22 //



Sensing Instrument

$m = 2.0$ kg
$k = 1.75$ kN/m

$2Y_{op} = 2.2$ mm
$2A = 2.0$ mm (double amplit. structure motion)

Y_g = vertical motion of supporting structure

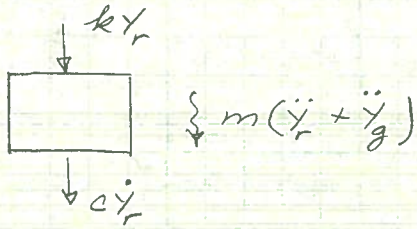
Y = TOTAL vertical motion of sensing instrument

Y_r = RELATIVE motion of sensing instrument $= Y - Y_g$

$Y_g = A \sin(\bar{\omega} t)$ mm $\qquad \bar{\omega} = 15\ Hz \left(\frac{2\pi\ rad}{cycle}\right) = 94.25\ rad/s$

$\ddot{Y} = -A\,\bar{\omega}^2 \sin(\bar{\omega} t)\ mm/s^2$

FBD - dynamic forces only



EoM

$$m\ddot{Y}_r + c\dot{Y}_r + kY_r = -m\ddot{Y}_g$$

$$m\ddot{Y}_r + c\dot{Y}_r + kY_r = mA\bar{\omega}^2 \sin(\bar{\omega} t)$$

Relative Displacement Response:

$$Y_r(t) = \underbrace{e^{-\zeta\omega t}\left[A\cos(\omega_d t) + B\sin(\omega_d t)\right]}_{\text{transient}} + \underbrace{\rho \sin(\bar{\omega} t - \phi)}_{\text{steady-state}}$$

From problem statement, given: $\rho = Y_{op} = 1.1$ mm

From previous derivation: $\rho = R_d\, \Delta_{ST}$

$$\Delta_{ST} = mA\bar{\omega}^2 / k$$

$$R_d = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \qquad \beta = \bar{\omega}/\omega$$

So for our system:

$$\omega = \sqrt{\frac{1.75\times10^3\ N/m}{2.0\ kg}} = 29.58\ rad/s$$

$$c_{cr} = 2m\omega = 2(2.0)(29.58) = 118.32\ N\text{-}s/m$$

$$\beta = \frac{\bar{\omega}}{\omega} = \frac{94.25}{29.58} = 3.1862$$

$$\Delta_{ST} = \frac{mA\bar{\omega}^2}{k} = \frac{(2.0\,kg)(0.001\,m)(94.25\ rad/s)^2}{1.75\times10^3\ N/m} = 1.015\times10^{-2}\,m$$

Then amplification factor R_d:

$$R_d = \rho/\Delta_{ST} = \frac{1.1\times10^{-3}\,m}{1.015\times10^{-2}\,m} = 0.10835$$

Substituting into eqn for R_d

$$R_d = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}$$

$$(1-\beta^2)^2 + (2\zeta\beta)^2 = \frac{1}{R_d^2}$$

$$\zeta^2 = \frac{1}{4\beta^2}\left(\frac{1}{R_d^2} - (1-\beta^2)^2\right)$$

$$\zeta^2 = \frac{1}{4(3.1862)^2}\left[\frac{1}{(1.0835\times10^{-1})^2} - (1-(3.1862)^2)^2\right]$$

$$\zeta^2 = \frac{1}{40.607}(85.177 - 83.757)$$

$$\zeta^2 = 3.498\times10^{-2}$$

$$\zeta = 0.187$$

To find damping coef:

$$c = \zeta c_{cr} = (0.187)(118.32) = \underline{\underline{22.13\ N\text{-}s/m}}$$

EXTRA: Plot steady-state response using both total & relative degrees of freedom. How do the answers compare?

For relative displacement:

$$Y_{r,ss}(t) = \rho \sin(\bar{\omega} t - \phi) \quad \text{where } \rho = 1.1 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta\beta}{1-\beta^2}\right) = \tan^{-1}\left(\frac{2(0.187)(3.1862)}{1-(3.1862)^2}\right)$$

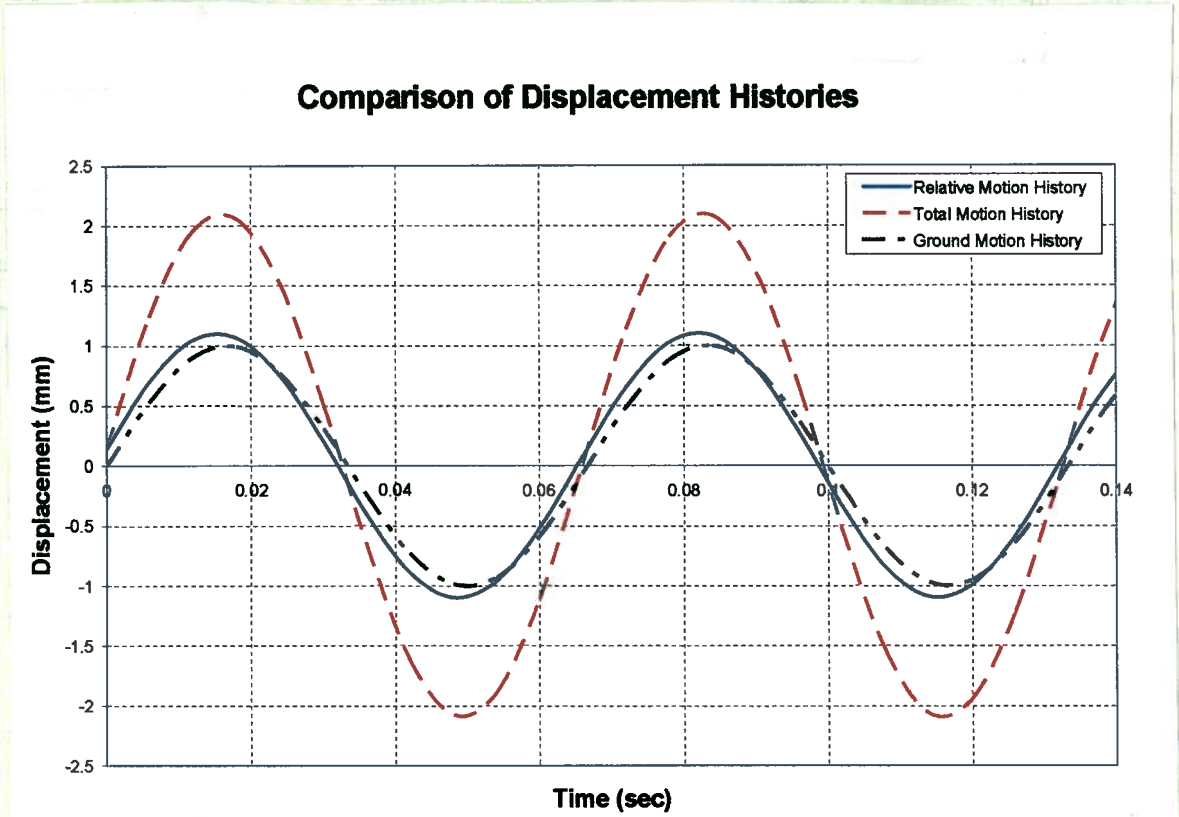
$$\phi = \tan^{-1}(-0.13021)$$

$$\phi = -0.12948$$

For total displacement

$$Y_{ss}(t) = Y_g(t) + Y_{r,ss}(t) = A\sin(\bar{\omega} t) + \rho \sin(\bar{\omega} t - \phi)$$

where $A = 1.0$ mm



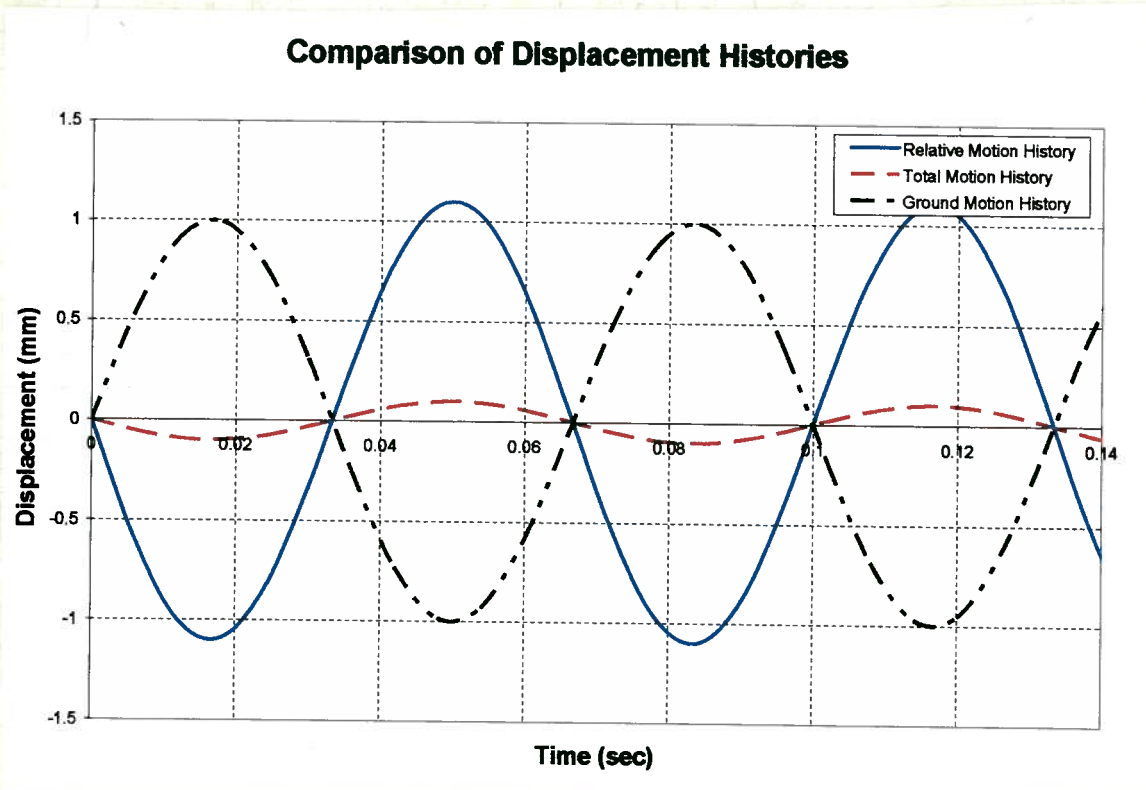
Is the amplitude of the total motion found by adding the amplitude of the ground motion & the amplitude of the relative motion?

From previous plot, amplitude of total motion appears to be 2.1 mm, which is the sum of the two amplitudes.

But notice that for this case the peak response for ground motion and relative motion occur @ almost the same time (very little phase lag).

What if these 2 components were out of phase?
ie- max for ground motion occurs @ same time as min for relative motion. ($\phi = \pi$ rad)

Then the response plots would look like:



Now you can clearly see that amplitude of total response is not the sum of the amplitudes of the two components.

Question: So how would you find peak response for total vibration?