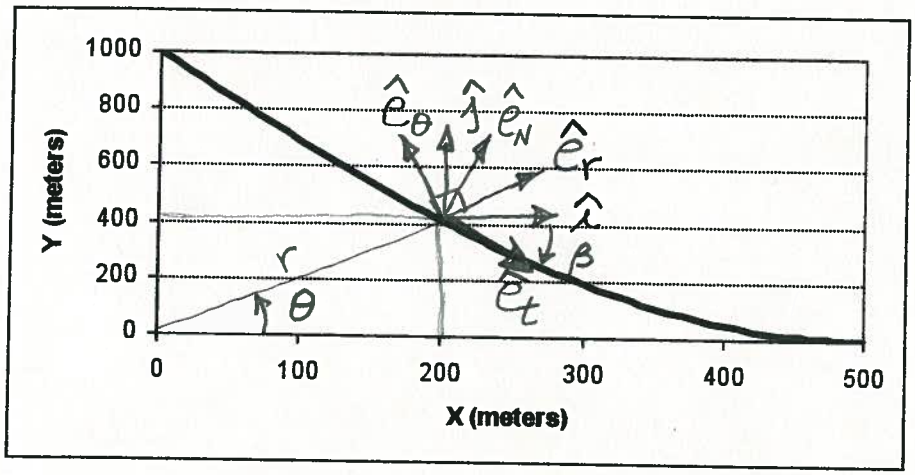
Problem 1 (40 points). Problem 1: A skier (particle P) slides down a mountain trail. The trail can be described with a path function $y(x)$ as given below:

$$y(x) = 1000\left[1 - \sin\left(\pi \frac{x}{1000}\right)\right] \quad \text{meters}$$

At $x = 200$ meters, the skier velocity is 50km/hr and his/her acceleration along the path is 0.5g.

(a) Calculate the skier's ***acceleration*** vector in all three coordinate systems: Cartesian, Polar, and Path. <u>You MUST draw the coordinate systems on the figure at the skier location for credit</u>. (30 pts)

(b) What is the force (expressed in your choice of Cartesian, Polar or Path) the trail exerts on the skier? Express your answer in vector format using appropriate unit vectors. (10 pts)



Need y' and y'' (Note: $' = \frac{d}{dx}$; $\dot{} = \frac{d}{dt}$)

$$y' = 1000\left[-\frac{\pi}{1000}\cos\left(\frac{\pi x}{1000}\right)\right] = -\pi\cos\left(\frac{\pi x}{1000}\right)$$

$$y'' = \frac{\pi^2}{1000}\sin\left(\frac{\pi x}{1000}\right)$$

@ $x = 200$ m $\quad y = 412.2$ m

$y' = -2.54$

$y'' = 0.0058$

$\tan\beta = \left|\frac{dy}{dx}\right| \Rightarrow \beta = 68.5^\circ$ <u>Note</u>: drawing is <u>NOT</u> to scale

$$\tan\theta = \frac{y}{x} \Rightarrow \theta = 64.1^\circ$$

$$\frac{1}{\rho} = \frac{|y''|}{\left(1+(y')^2\right)^{3/2}} = 2.85 * 10^{-4}$$

$$v = 50\ \text{km/hr} * \frac{1000\text{m}}{\text{km}} * \frac{\text{hr}}{3600\text{s}} = 13.89\ \text{m/s}$$

$$a_T = 0.5(9.8\ \text{m/s}^2) = 4.9\ \text{m/s}^2$$

$$|a| \approx 4.9\ \text{m/s}^2$$

$$a_N = \frac{v^2}{\rho} = (13.89)^2(2.85*10^{-4}) = 0.055\ \text{m/s}^2$$

Transforming <u>Path</u> to <u>Cartesian</u> $\Rightarrow 68.5^\circ$ c.c.w.

$$\begin{Bmatrix} a_x \\ a_y \end{Bmatrix} = \begin{bmatrix} \cos(68.5) & \sin(68.5) \\ -\sin(68.5) & \cos(68.5) \end{bmatrix} \begin{Bmatrix} 4.9 \\ 0.055 \end{Bmatrix} = \begin{Bmatrix} 1.85\ \text{m/s}^2 \\ -4.54\ \text{m/s}^2 \end{Bmatrix}$$

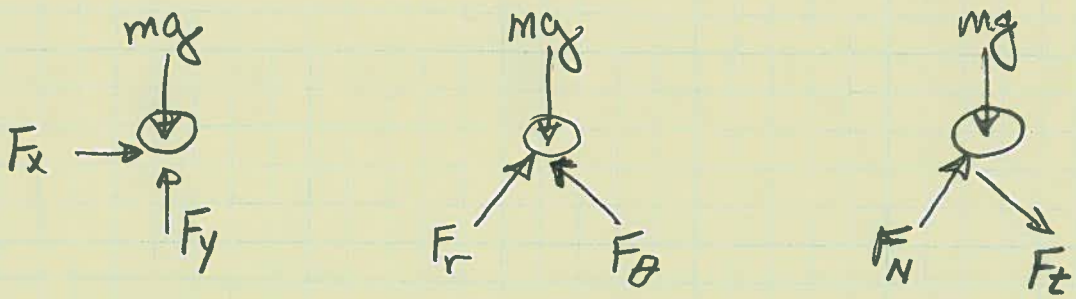
check $|a| = \sqrt{(1.85)^2 + (4.54)^2} \approx 4.9\ \text{m/s}^2$ ✓

Transforming <u>Cartesian</u> to <u>Polar</u> $\Rightarrow 64.1^\circ$ CCW

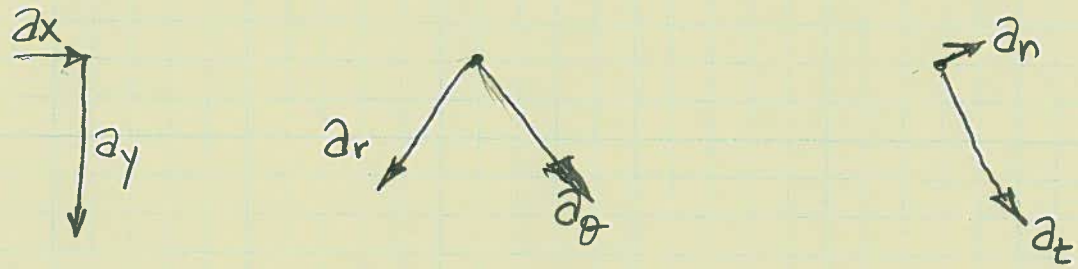
$$\begin{Bmatrix} a_r \\ a_\theta \end{Bmatrix} = \begin{bmatrix} \cos(64.1) & \sin(64.1) \\ -\sin(64.1) & \cos(64.1) \end{bmatrix} \begin{Bmatrix} 1.85 \\ -4.54 \end{Bmatrix} = \begin{Bmatrix} -3.28\ \text{m/s}^2 \\ -3.65\ \text{m/s}^2 \end{Bmatrix}$$

check $|a| = \sqrt{(3.28)^2 + (3.65)^2} \approx 4.9\ \text{m/s}^2$ ✓

CARTESIAN Polar Path



first check accelerations for "reasonableness"



all appear down to right
(and cry out for "scale" & "protractor")

For CARTESIAN F.B.D. above

$$\overrightarrow{\Sigma F_x} = m a_x = \underline{1.85m} \text{ (N)}$$

$$\uparrow \Sigma F_y = m a_y$$

$$F_y - mg = m(-4.54)$$

$$F_y = (9.8 - 4.54)m = \underline{5.26m} \text{ (N)}$$

$$\{F\} = \begin{Bmatrix} 1.85m \\ 5.26m \end{Bmatrix} N$$

$$\vec{F} = 1.85m\,\hat{i} + 5.26m\,\hat{j}$$