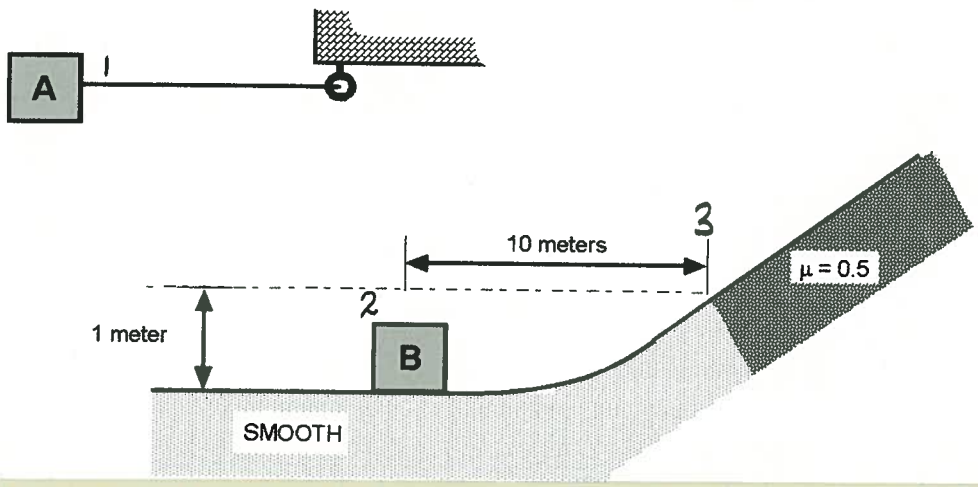
Problem 3: (25 points) A **20-kg** block (Block *A*) is suspended from a **9 meter** rope. It swings down as shown in the figure below and strikes a **10 kg** block (Block *B*). The blocks collide when the rope is oriented perfectly downwards and is perfectly straight. The coefficient of restitution between the block has been determined to be **0.6**. Block B then travels horizontally along a smooth (frictionless) surface and eventually travels up a slope that becomes rough. The angle the slope makes with the horizontal is 30 degrees..

Tasks:

a. How long (after block enters the slope) will it be before block *B* comes to rest? (Note: you don't have to worry about whether or not the block will start sliding back down the slope.)

b. How far will it have traveled from the start of the rough section of the slope?



PHASE I ⇒ Block A swings from horizontal position to "just before" impact with Block B

Conservation of Energy ⇒ $mgh_1 = \frac{1}{2} m v_2^2$

$$v_2 = \sqrt{2gh}$$

$$v_2 = \sqrt{2(9.8\,\tfrac{m}{s^2})(9\,m)}$$

$$= 13.28\ m/s$$

PHASE II impact

Conservation of Momentum

$$m_A v_2 + 0 = m_A v_2' + m_B v_B$$

$$20\,kg(13.28\,m/s) = 20 v_2' + 10 v_B$$

$$26.56 = 2 v_2' + v_B \quad \Leftarrow \text{EQN 1}$$

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coefficient of restitution $= 0.6 = \dfrac{v_B - v_2'}{13.28}$

$$7.97 = v_B - v_2' \Leftarrow \text{EQN 2}$$

substituting EQN 2 into EQN 1

$$26.56 = 2(v_B - 7.97) + v_B$$

$$42.5 = 3v_B$$

$$14.17\ \text{m/s} = v_B$$

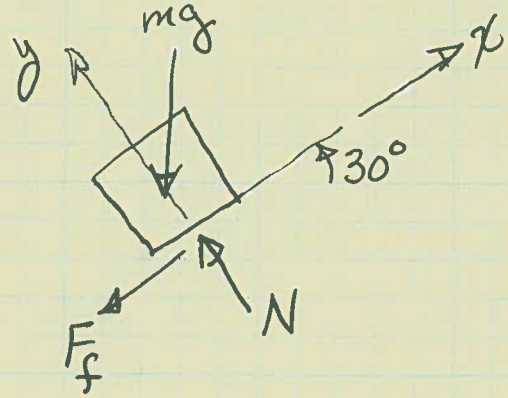
PHASE III block B slides $2 \rightarrow 3$

Conservation of Energy $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_B'^2 + mgh$

$$\tfrac{1}{2}(\cancel{10}\,\text{kg})(14.17\ \text{m/s})^2 = \tfrac{1}{2}(\cancel{10}\,\text{kg})v_B'^2 + (\cancel{10}\,\text{kg})(9.8\ \text{m/s}^2)(1\ \text{m})$$

$$\sqrt{2(100.39 - 9.8)} = v_B' = 13.46\ \text{m/s}$$

PHASE IV WORK & ENERGY & (de)acceleration



$$\Sigma F_y = ma_y = 0$$

$$N - mg\cos 30° = 0$$

$$N = \underline{\underline{84.87\ N}}$$

$$\Sigma F_x = ma_x$$

$$-mg\sin 30° - \mu N = ma_x$$

$$-91.44\ N = (10\ \text{kg})a_x$$

$$-9.14\ \text{m/s}^2 = a_x$$

a) $v_i = 13.46\ m/s$; $a = -9.14\ m/s^2$

Kinematics (for constant acceleration)

$$v_f = v_i + at$$

$$0 = 13.46 - 9.14t \longrightarrow t_{stop} = \underline{\underline{1.47 \text{ seconds}}}$$

<u>Note</u>: could have started from $a = \frac{dv}{dt}$

$$\int a\,dt = \int dv$$

$$a(\Delta t) = \Delta v = v - v_i$$

b) distance could come from kinematics

$$v = \frac{dx}{dt} \Rightarrow \int v\,dt = \int dx$$

$$\int (v_i + at)\,dt = x - x_i$$

$$\cancel{x_i}^{\,0} + v_i(\Delta t) + \tfrac{1}{2}\overset{-9.14\ m/s^2}{a}t^2 = x = \underline{\underline{9.91\,m}}$$

$13.46\ m/s$ $\quad$ $1.47\,s$

-OR-

could find from work energy

$$\tfrac{1}{2}m v_3^2 = \underbrace{mgh + \text{Work}}_{} = \boxed{91.44\,N * \text{distance}}$$

Note: Cannot count gravity twice (use $F_f = 42.44\,N$ for work if include mgh)

$$\frac{1}{2}(10)(13.46)^2 = mgh + \text{Work}$$

$$905.86 = 10(9.8)(d \sin 30^\circ) + 42.44 * d$$

$$\frac{905.86}{(49+42.44)} = d = 9.91\,m$$

-OR-

$$\frac{1}{2}(10)(13.46)^2 = 91.44 * d$$

$$\frac{905.86}{91.44} = d = 9.91\,m$$

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