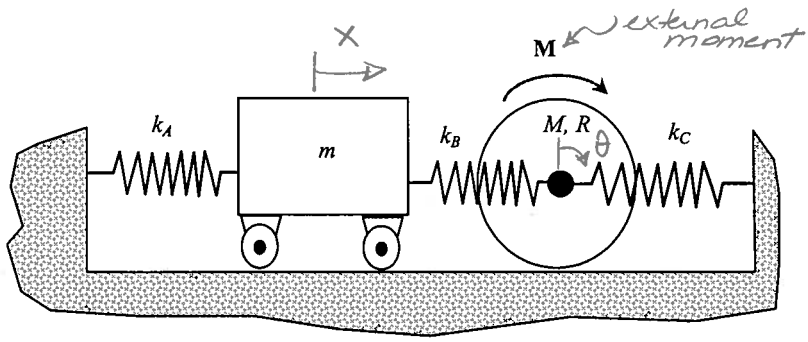
Problem 3: The cylinder of mass M and radius R rolls with out slipping and is attached to the cart of mass m by spring k_B. **Tasks:**
(a) Draw the free body diagram for each body.
(b) Obtain the equation of motion for each body. Present the equations in matrix format.



DOF

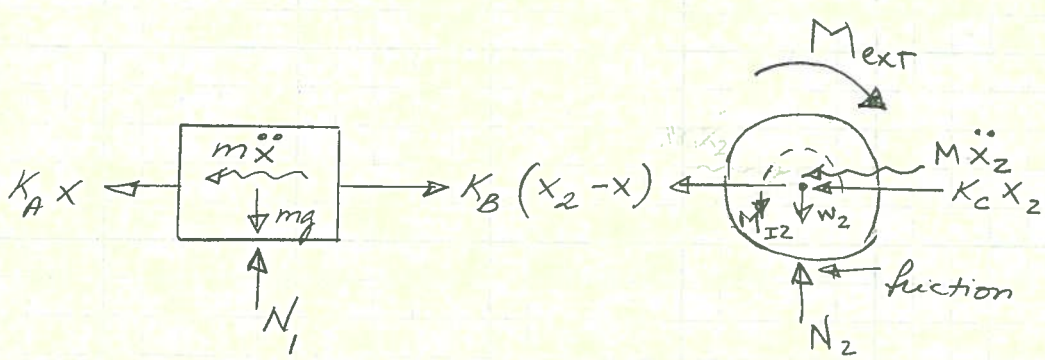
X = total horizontal motion of cart from s.e.

θ = CW rotation of disk from s.e. (total)

Other: x_2 = horiz motion of CM of disk

Kinematics: $x_2 = R\theta$

FBD - Total



$$M_{I2} = \left(\frac{MR^2}{2}\right)\ddot{\theta}$$

Note: assuming springs @ height of CM of cart!

Apply equilibrium: cart

$$m\ddot{x} + K_A x - K_B(x_2 - x) = 0$$

$$m\ddot{x} + (K_A + K_B)x - (K_B R)\alpha = 0$$

Apply equilibrium: disk

$$(M\ddot{x}_2)R + M_{I2} + (K_c x_2)R + K_B(x_2 - x)R - M_{ext} = 0$$

$$(MR^2)\ddot{\theta} + \left(\frac{MR^2}{2}\right)\ddot{\theta} + (K_c R^2)\theta + K_B(R^2\theta - Rx) - M_{ext} = 0$$

$$\left(\frac{3MR^2}{2}\right)\ddot{\theta} + (R^2(K_c + K_B))\theta - (K_B R)x = M_{ext}$$

In matrix form

$$\begin{bmatrix} m & 0 \\ 0 & \frac{3MR^2}{2} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} (K_A + K_B) & -K_B R \\ -K_B R & R^2(K_c + K_B) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_{ext} \end{Bmatrix}$$