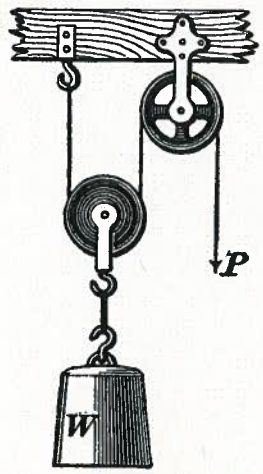
Problem 2: A suspended object has a weight, W, of 25 lbs. The pulley directly over the object has a radius of 4 inches and weighs 5 lbs. The other pulley also has a radius of 4 inches and weighs 3 lbs. Both pulleys can be treated as disks with uniform mass. If the force on the inextensible rope is 25 lbs:

(a) What is the magnitude of the acceleration of the suspended object?
(b) How far will it have traveled in 1 second if it starts from rest?
(c) What force would be required (instead of the 25lbs) for the object to travel upwards 1 ft in 1 second?



DOF: y = vertical motion of suspended object from position when at rest

Other Variables:

θ_A = Rotation of pulley A $\circlearrowleft +$

θ_B = Rotation of pulley B $\circlearrowleft +$

R = Radius of pulleys A & B = 4 in

m_W = mass of object $= \dfrac{25 \text{ lbs}}{386.4 \text{ in/s}^2} = 0.06470 \text{ lb-s}^2/\text{in}$

m_A = mass pulley A $= \dfrac{5 \text{ lbs}}{386.4 \text{ in/s}^2} = 0.01294 \text{ lb-s}^2/\text{in}$

m_B = mass pulley B $= \dfrac{3 \text{ lbs}}{386.4 \text{ in/s}^2} = 0.007764 \text{ lb-s}^2/\text{in}$

Kinematics: $y = R\theta_A$

$\ddot{y} = R\ddot{\theta}_A \rightarrow \ddot{\theta}_A = \ddot{y}/R$

$$\{a_1\} = \underbrace{\ddot{y}\{j\}}_{\text{center pulley}} + \underbrace{\ddot{\theta}_A\{k\} \times R\{i\}}_{\text{tangential}} + \underbrace{\dot{\theta}_A\{k\} \times \left(\dot{\theta}_A\{k\} \times R\{i\}\right)}_{\text{normal}}$$

$$\{a_2\} = \underbrace{0\{i\} + 0\{j\}}_{\text{center pulley}} + \underbrace{\ddot{\theta}_B\{k\} \times (-R)\{i\}}_{\text{tangent}} + \underbrace{\dot{\theta}_B\{k\} \times \left(\dot{\theta}_B\{k\} \times (-R)\{i\}\right)}_{\text{normal}}$$

Since only tangential components 'transfer'/match

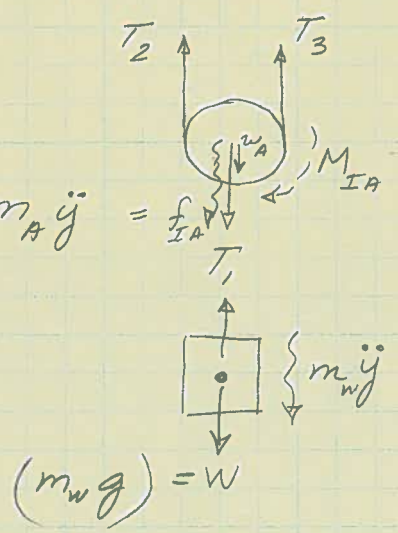
$$a_{1,y} = a_{2,y}$$

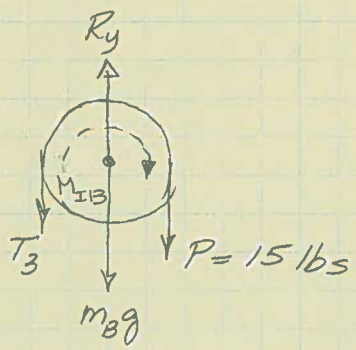
$$\ddot{y}\{j\} + (R\ddot{\theta}_A)\{j\} = 0\{j\} - R\ddot{\theta}_B\{j\}$$

$$\left[(R\ddot{\theta}_A) + (R\ddot{\theta}_A)\right]\{j\} = (-R\ddot{\theta}_B)\{j\}$$

$$\ddot{\theta}_B = -2\ddot{\theta}_A = -2\ddot{y}/R$$

<u>FBD</u>:





$$M_{IA} = \left(\frac{m_A R^2}{2}\right)\ddot{\theta}_A$$

$$M_{IB} = \left(\frac{m_B R^2}{2}\right)\ddot{\theta}_B$$

Apply equilibrium:

① $m_W \ddot{y} + m_W g - T_1 = 0$ $\qquad \Sigma F_y = 0$ for block

② $m_A \ddot{y} + m_A g + T_1 - T_2 - T_3 = 0$ $\qquad \Sigma F_y = 0$ for pulley A

③ $(m_A \ddot{y})R + T_1 R + (m_A g)R + M_{IA} - T_3(2R) = 0$

Combining ① and ③

$$(m_A \ddot{y})R + (m_W \ddot{y})R + (m_A + m_W)gR + M_{IA} - T_3(2R) = 0$$

④ $(m_A + m_W)\ddot{y}R + (m_A + m_W)gR + \left(\frac{m_A R^2}{2}\right)\ddot{\theta}_A - T_3(2R) = 0$

Looking @ pulley B

$$T_3 R - M_{IB} - PR = 0$$

⑤ $T_3(2R) - \cancel{2}\left(\frac{m_B R^2}{\cancel{2}}\right)\ddot{\theta}_B - (2P)R = 0$

Combining ④ & ⑤

$$(m_A + m_W)\ddot{y}\cancel{R} + (m_A + m_W)g\cancel{R} + \left(\frac{m_A R^{\cancel{2}}}{2}\right)\ddot{\theta}_A - (m_B R^{\cancel{2}})\ddot{\theta}_B - (2P)\cancel{R} = 0$$

$$(m_A + m_W)\ddot{y} + (m_A + m_W)g + \left(\frac{m_A \cancel{R}}{2}\right)\left(\frac{\ddot{y}}{\cancel{R}}\right) - (m_B \cancel{R})\left(\frac{-2\ddot{y}}{\cancel{R}}\right) = 2P$$

$$(m_W + 1.5 m_A + 2 m_B)\ddot{y} = 2P - (m_A + m_W)g$$

Substituting values & solving for P:

$$\ddot{y} = \frac{[2(25) - (25+5)](386.4)}{(25 + (1.5)(5) + (2)(3)]} = 200.73 \text{ in}/s^2$$

b) $\dot{y} = \int_0^t \ddot{y}\, dt = \ddot{y}t + \dot{y}_0$

$y = \int_0^t \dot{y}\, dt = \int_0^t (\ddot{y}t + \dot{y}_0)\, dt$

$y = y_0 + \cancel{\dot{y}_0}^{\,0,\text{ starts @ rest}} t + \frac{1}{2}\ddot{y}t^2$

$(y - y_0) = \frac{1}{2}(200.73)(1\,\text{sec})^2$

$= 100.36$ inches

$= 8.36$ ft

c) What constant force required to travel 1 ft in 1 second?

$1\text{ ft} = \frac{1}{2}\ddot{y}(1\,\text{sec})^2$

$12\text{ in} = \frac{1}{2}\ddot{y}(1\,\text{sec})^2$

$\ddot{y} = 24\text{ in}/s^2$

From previous eqn:

$2P = (m_W + 1.5m_A + 2m_B)\ddot{y} + (m_A + m_W)g$

$P = \frac{1}{2}\left[\left(\frac{25 + 1.5(5) + 2(3)}{386.4\text{ in}/s^2}\right)(24\text{ in}/s^2) + (25^{\#} + 5^{\#})\right]$

$P = 16.18$ lbs