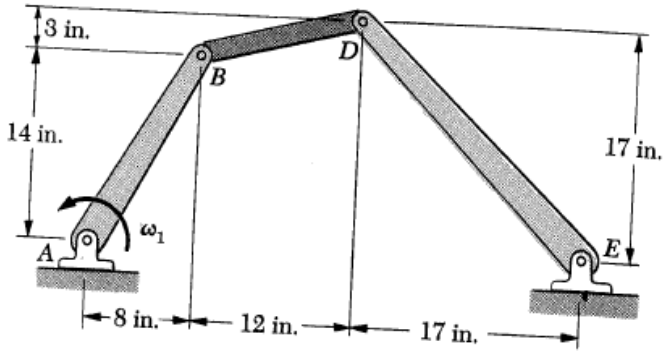
Problem 1: The linkage *ABDE* moves in the vertical plane. Knowing that in the position shown crank *AB* has a constant angular velocity ω_1 of 20 rad/sec counterclockwise, determine the angular velocities and accelerations of the connecting rod *BD* and of the crank *DE*. Use vector equations to solve.



Problem 1//

$\{\omega_{AB}\} = 20\{k\}$ rad/s $\qquad \{\alpha_{AB}\} = 0$

Velocities

① $\{V_B\} = \cancel{\{V_A\}}^{0} + \{\omega_{AB}\} \times \{r_{AB}\}$

$= 20\{k\} \times (8\{i\} + 14\{j\})$

$= -280\{i\} + 160\{j\}$

② $\{V_D\} = \{V_B\} + \{V_{D|B}\} = \{V_B\} + \{\omega_{BD}\}\{k\} \times \{r_{DB}\}$

③ $\{V_D\} = \cancel{\{V_E\}}^{0} + \{\omega_{ED}\} \times \{r_{ED}\}$

where: $\{r_{DB}\} = 12\{i\} + 3\{j\}$

$\{r_{ED}\} = -17\{i\} + 17\{j\}$

$\{\omega_{BD}\} = \omega_{BD}\{k\}$

$\{\omega_{ED}\} = \omega_{ED}\{k\}$

Subst. ① into ② & equating result to ③

$(-280\{i\} + 160\{j\}) + \omega_{BD}\{k\} \times (12\{i\} + 3\{j\})$

$= \omega_{ED}\{k\} \times (-17\{i\} + 17\{j\})$

$(-280\{i\} + 160\{j\}) + (-3\omega_{BD}\{i\} + 12\omega_{BD}\{j\})$

$= (-17\omega_{ED}\{i\} - 17\omega_{ED}\{j\})$

Coeff. of $\{i\}$

$-280 - 3\omega_{BD} = -17\omega_{ED}$

Coeff of $\{j\}$

$+160 + 12\omega_{BD} = -17\omega_{ED}$

Setting them equal:

$$-280 - 3\omega_{BD} = 160 + 12\omega_{BD}$$

$$-440 = 15\omega_{BD}$$

$$\underline{\omega_{BD} = -29.33 \text{ rad/sec}}$$

So:

$$17\omega_{ED} = 280 - 3(29.33)$$

$$\underline{\omega_{ED} = 11.29 \text{ rad/s}}$$

So:

$$\boxed{\begin{aligned} \{\omega_{BD}\} &= -29.33\{K\} \text{ rad/sec} \\ \{\omega_{ED}\} &= 11.29\{K\} \text{ rad/sec} \end{aligned}}$$

<u>Accelerations</u>

$$\{\alpha\}_{BD} = \alpha_{BD}\{K\}$$

$$\{\alpha_{DE}\} = \alpha_{DE}\{K\}$$

$$\{a_B\} = \cancelto{0}{\{\alpha_{AB}\}} \times \{r_{AB}\} + \{\omega_{AB}\} \times (\{\omega_{AB}\} \times \{r_{AB}\})$$

$$\{a_D\} = \{a_B\} + \overbrace{\{\alpha_{BD}\} \times \{r_{BD}\} + \{\omega_{BD}\} \times (\{\omega_{BD}\} \times \{r_{BD}\})}^{\{a_{D|B}\}}$$

$$\{a_D\} = \cancelto{0}{\{a_E\}} + \{\alpha_{DE}\} \times \{r_{DE}\} + \{\omega_{DE}\} \times (\{\omega_{DE}\} \times \{r_{DE}\})$$

$$\{a_B\} = (20)^2(8i + 14j) = -3200\{i\} - 5600\{j\}$$

$$\{a_{D|B}\} = \{\alpha_{BD}\} \times \{r_{BD}\} + \{\omega_{BD}\} \times (\{\omega_{BD}\} \times \{r_{BD}\})$$

$$= (12\alpha_{BD}\{j\} - 3\alpha_{BD}\{i\}) + (-10{,}320\{i\} - 2580\{j\})$$

$$\{a_{D|E}\} = \{\alpha_{DE}\} \times \{r_{DE}\} + \{\omega_{DE}\} \times (\{\omega_{DE}\} \times \{r_{DE}\})$$

$$= (-17\alpha_{DE}\{j\} - 17\alpha_{DE}\{i\}) + (2170\{i\} - 2170\{j\})$$

Combining Eqns & equating coef. of {i} & {j}

$$\begin{cases} -17\alpha_{DE} + 3\alpha_{BD} = -15,690 \\ -17\alpha_{DE} - 12\alpha_{BD} = -6010 \end{cases}$$

Solving:

$$\boxed{\begin{aligned} \alpha_{BD} &= -645\{k\} \text{ rad/sec} \\ \alpha_{DE} &= 809\{k\} \text{ rad/sec} \end{aligned}}$$