Problem: (Based on Old Exam Problem). Two blocks, A and B, are mounted on a smooth vertical track as shown in the Figure 1 below. Block A weighs 15 lbs while block B weighs 10 lbs. Block B also has a spring support at shown, with the spring being undeformed when it is oriented perfectly horizontally as in the figure 1. Blocks A is to be released from rest when in the position shown in figure 2, with block A at a position of 2 feet above block B, which is at rest in its the static equilibrium position. When block A hits block B, the two blocks stick together. If the spring stiffness is k = 100 lb/ft, what is:

a) the maximum stretch of the spring when the two masses first stop moving downwards
b) the position of the two masses when they first stop moving downwards

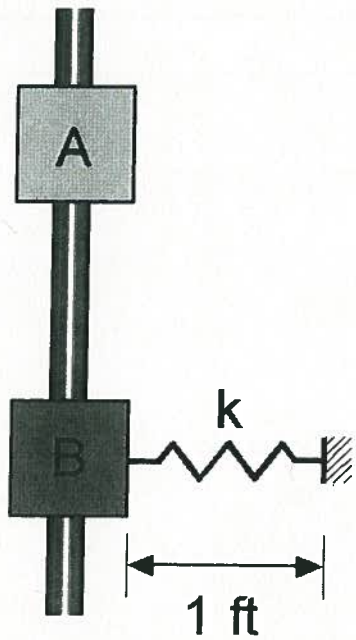


Figure 1

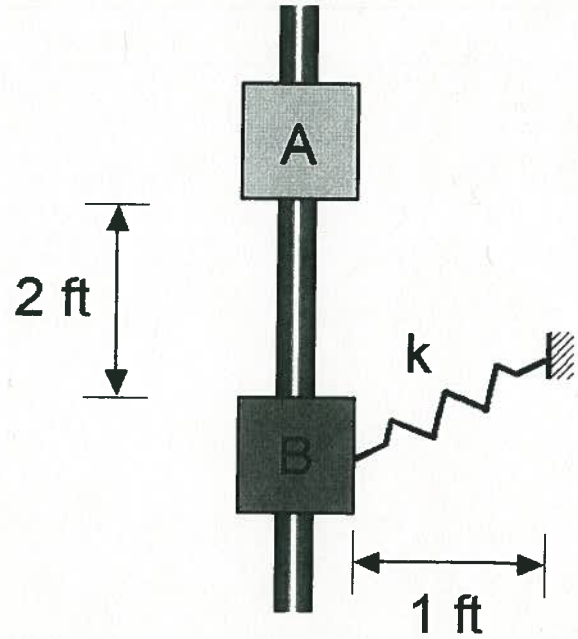
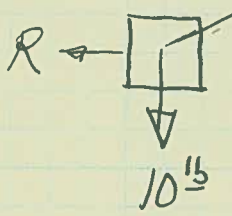


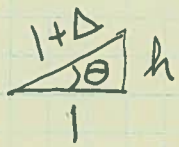
Figure 2

Calculation static equilibrium position



$F_S = K\Delta = 100 \frac{lb}{ft} \cdot \Delta$

$\uparrow \Sigma F = 0 \Rightarrow$ vertical component of $F_S = 10^{lb} \Rightarrow F_S \sin\theta = 10^{lb}$



$\sin\theta = \frac{h}{1+\Delta} = \frac{\sqrt{\Delta^2+2\Delta}}{1+\Delta}$

$\cos\theta = \frac{1}{1+\Delta}$

$1^2 + h^2 = (1+\Delta)^2 \Rightarrow h = \sqrt{\Delta^2 + 2\Delta}$

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$$100 \cdot \Delta \left(\frac{\sqrt{\Delta^2 + 2\Delta}}{1+\Delta} \right) = 10^{\frac{1}{6}}$$

$$\Delta^2 + 2\Delta = \left(\frac{10(1+\Delta)}{100\Delta} \right)^2$$

$$(\Delta^2 + 2\Delta)(10000\Delta^2) = 100(1+\Delta)^2$$

$$10000\Delta^4 + 20000\Delta^3 = 100(1 + 2\Delta + \Delta^2)$$

$$10000\Delta^4 + 20000\Delta^3 - 100\Delta^2 - 200\Delta - 100 = 0$$

$$100\Delta^4 + 200\Delta^3 - \Delta^2 - 2\Delta - 1 = 0$$

Δ	RHS
0.1	-1
0.2	$+0.32$
0.175	-0.215
0.185	-0.0208
0.186	E-05 ≈ 0

$\Delta = 0.186$ ft

& from page 1

$h = 0.6376$ ft

Note: of course this fourth order equation could be solved in a more elegant fashion, but ...

v_A before impact

$$v_A = \cancel{v_0} + at$$

$$2ft = y_A = \cancel{y_0} + \cancel{v_0} t + \tfrac{1}{2} a t^2$$

since $a = 32.2\ ft/s^2$, can solve for v_A & t

$$2^{ft} = \tfrac{1}{2}(32.2)t^2 \Rightarrow t = 0.3525s$$

$$v_A = 32.2(0.3525) = 11.35\ ft/s$$

also could use conservation of energy for mass A

Conservation of momentum (and plastic impact)

$v_{A\&B}$ AFTER impact

$$m_A v_A + m_B \cancel{v_B}^{0} = (m_A + m_B) v_{A\&B}$$

$$\frac{15}{32.2}(11.35) = \frac{25}{32.2} v_{A\&B}$$

$$v_{A\&B} = 6.81\ ft/s$$

Energy "just after impact" $= \frac{1}{2} m v^2 + mgh + \frac{1}{2} k \Delta^2$

depends on our choice of reference

IF WE CHOOSE horizontal position of spring as the reference

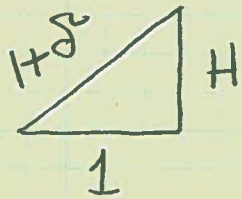
$$E_i = \frac{1}{2}\left(\frac{25}{32.2}\right)(6.81)^2 - 25h + \frac{1}{2}(100)\Delta^2$$

$$= 3.79\ ft \cdot lb$$

see page 2 $\begin{cases} \Delta = 0.186^{ft} \\ h = 0.6376^{ft} \end{cases}$

$$E_f = -25H + \frac{1}{2}(100)\delta^2$$

see page 4



looks just like figure on the bottom of page 1 ∴ all basic relationships developed for that case still hold (substituting H for h and δ for Δ) <u>EXCEPT</u> that $F_{spring} \neq 10^{16}/\sin\theta$

$$H = \sqrt{\delta^2 + 2\delta^2}$$

& $$E_f = -25H + \tfrac{1}{2}(100)\delta^2 = -25\sqrt{\delta^2+2\delta} + \tfrac{1}{2}(100)\delta^2$$

<u>Note</u>: δ = answer the part a) & H = answer to part b)

$$3.79 = -25\sqrt{\delta^2+2\delta} + 50\delta^2$$

δ	LHS
1	6.699
0.95	3.273
0.96	3.937
0.958	3.804
0.9575	3.77
0.9578	3.79 ✓

$\delta = 0.9578$ ft ⇐ a)

$H = 1.683$ ft ⇐ b)