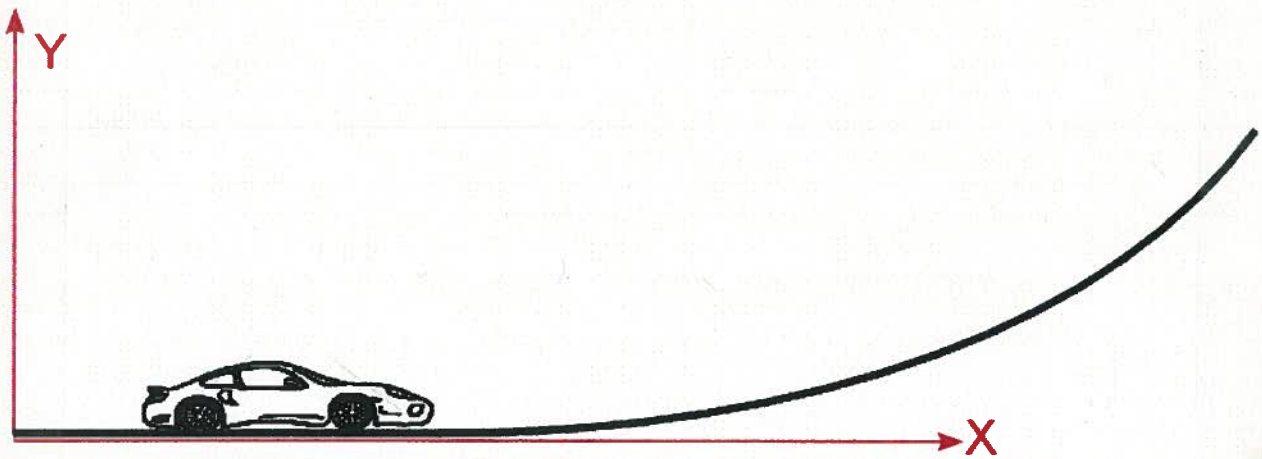
Problem 2. A car travels at a constant speed of 100 km/hr on a smooth road of increasing grade whose vertical profile can be approximated by the equation:

$$y = 0.0003x^2$$

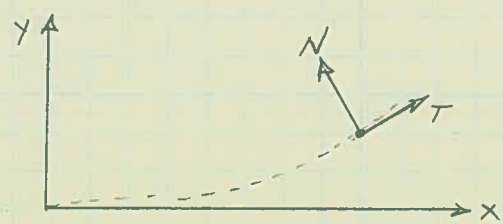
given the coordinate system shown when x is measured in meters and the vertical height y is also in meters. When the horizontal coordinate for the car is x = 400 meters, what are the tangential and normal components of the car's acceleration?



Solution

$y = 0.003x^2$

@ $x = 400m \longrightarrow y = 0.003(400)^2 = 48\,m$



$\{a\} = a_T\{e_T\} + a_N\{e_N\}$

$\underline{a_T = 0}$ since constant speed

$a_N = \dfrac{v^2}{\rho}$ where $\dfrac{1}{\rho} = \dfrac{|y''|}{[1+(y')^2]^{3/2}}$

$v = \left(100\,\dfrac{Km}{hr}\right)\left(\dfrac{hr}{3600s}\right)\left(\dfrac{1000\,m}{Km}\right) = 27.778\ m/s$

$y' = \dfrac{dy}{dx} = 0.006x$ @ $x = 400m$ $y' = 2.4\ m/m$

$y'' = \dfrac{d^2y}{dx} = 0.006$ @ $x = 400\,m$ $y'' = 0.006\ m/m^2$

$$\frac{1}{\rho} = \frac{|0.006|}{\left[1 + (2.4)^2\right]^{3/2}} = 3.4137 * 10^{-4} \ m^{-1}$$

$$a_N = (27.778 \ m/s)^2 (3.4137 * 10^{-4})$$

$$= \underline{\underline{0.2634 \ m/s^2}}$$

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