1) A vehicle is moving along a path defined by:

$$Y_b = A\sin\left(\frac{2\pi X}{L}\right),$$

where A = 0.03 m, L = 0.3 m. The vehicle has a constant velocity

$$v = \dot{X} = v_0 \quad {}^{km}/_{hr}$$

For convenience, assume that it starts at x = 0, then the position of the vehicle can be defined by:

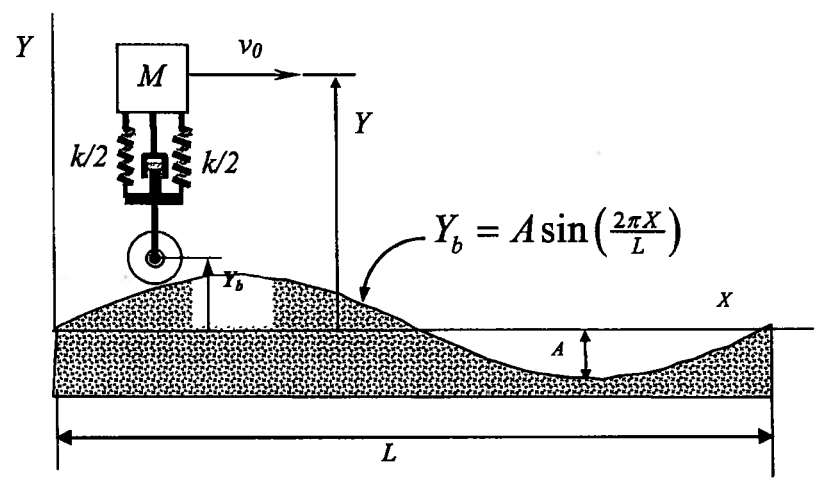
$$X = X_0 + \dot{X}t = 0 + v_0\left(\frac{km}{hr}\right)\left(\frac{1hr}{3600\ sec}\right)\left(\frac{1000\ m}{1\ km}\right)t \Rightarrow X = 0.278v_0 t \quad m$$

Tests show that the vehicle has a natural frequency of 5 Hz (cycles/sec). The mass of the vehicle is 750 kg.

Tasks:

(a). Approximately what speed will cause the highest steady-state amplitude for the vehicle?

(b). Select the a damping value c for the peak running speed in question a such that the vehicle's steady-state amplitude will be less than 0.12m.



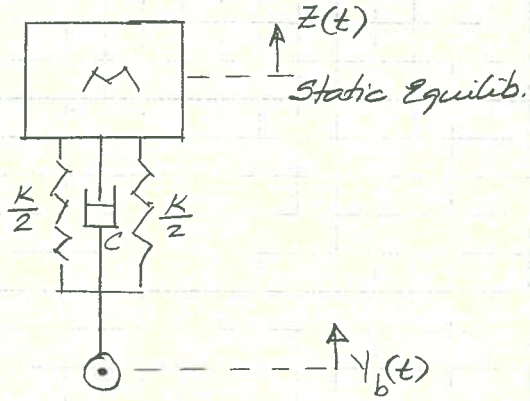
From car's perspective, ground is moving up and down underneath it.

Note that equations Y and X are given with respect to fixed coordinate system in figure.

So these variables describe absolute motion

Variables Y = absolute vertical displacement measured from centerline of ground

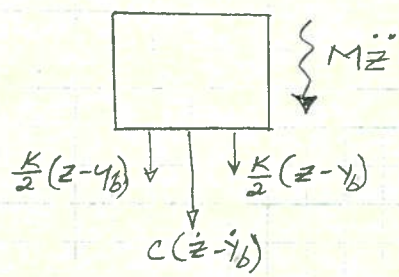
Z = absolute vertical motion measured from s.e. position



to use y would need length of springs.

<u>DOF</u> : z as defined above

<u>FBD - IDF</u>



<u>Equilibrium</u>

$$M\ddot{z} + k(z - y_b) + c(\dot{z} - \dot{y}_b) = 0$$

$$\boxed{M\ddot{z} + c\dot{z} + kz = ky_b + c\dot{y}_b}$$

Need to express y_b as function of time:

$$y_b(x) = A\sin\left(\frac{2\pi x}{L}\right) \quad \text{and} \quad x(t) = 0.278 v_0 t$$

$$y_b(t) = A\sin\left(\frac{2\pi}{L}(0.278 v_0 t)\right) = A\sin\left(\frac{1.747 v_0 t}{L}\right)$$

Given: $A = 0.03\,m$, $L = 0.3\,m$

$$y_b(t) = 0.03\sin(5.822 v_0 t)$$

$$\dot{y}_b(t) = (0.1747 v_0)\cos(5.822 v_0 t)$$

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So EoM:

$$\boxed{M\ddot{z} + c\dot{z} + Kz = [K(0.03)]\sin(5.822 v_0 t) + [c(0.1747 v_0)]\cos(5.822 v_0 t)}$$

a) Speed that will cause highest steady-state amplitude: $\beta \approx 1$. Assuming small damping then take $\beta = 1$ (resonance)

So need to find when $\frac{\bar{\omega}}{\omega} = 1$

$$\bar{\omega} = \omega$$

ω = undamped natural frequency $= \sqrt{\frac{k^*}{m^*}}$

Given: vehicle has nat. freq = 5 Hz

$$\omega = \left(5 \frac{\text{cycles}}{\text{s}}\right)\left(\frac{2\pi \text{ rad}}{\text{cycle}}\right) = 31.416 \text{ rad/s}$$

From EoM: $\bar{\omega} = 5.822 v_0$

So: $5.822 v_0 = 31.416$

$$v_0 = 5.396 \text{ Km/hr}$$ (see units in prob. statement)

b) Select damping const c w/ speed as in (a) such that peak s.s. amplitude < 0.12m

EoM is of form:

$$M\ddot{z} + c\dot{z} + Kz = f_{01}\sin(\bar{\omega}t) + f_{02}\cos(\bar{\omega}t)$$

$$f_{01} = 0.03K$$

$$f_{02} = 0.1747 v_0 c = 0.1747(5.396)c = 0.9427c$$

$$\bar{\omega} = 5.822 v_0 = 5.822(5.396) = 31.416 \text{ rad/s}$$

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Steady-state Response → 2 components since 2 forcing functions

$$z_{ss}(t) = \underbrace{\rho_1 \sin(\bar{\omega}t - \phi_1)}_{\text{due to } f_{01}\sin(\bar{\omega}t)} + \underbrace{\rho_2 \cos(\bar{\omega}t - \phi_2)}_{\text{due to } f_{02}\cos(\bar{\omega}t)} \quad (1)$$

Since steady-state is summation of SIN and COS functions of same frequency $\bar{\omega}$, then can be written as:

$$z_{ss}(t) = \underbrace{X}_{\text{Amplitude of S.S.}} \sin(\bar{\omega}t + \gamma) \quad (2)$$

where $X \neq \rho_1 \neq \rho_2$

$\gamma \neq \phi_1 \neq \phi_2$ in general.

For each particular:

$$\rho_1 = \left(\frac{f_{01}}{K}\right)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right) = (0.03)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)$$

$$\rho_2 = \left(\frac{f_{02}}{K}\right)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right) = \frac{0.9427c}{K}\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)$$

$$= (0.03)\left(\frac{c\bar{\omega}}{K}\right)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)$$

(3) So: $$z_{ss}(t) = (0.03)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)\left[\sin(\bar{\omega}t - \phi_1) + \frac{c\bar{\omega}}{K}\cos(\bar{\omega}t - \phi_2)\right]$$

But in our case: $\phi_1 = \phi_2 = \tan^{-1}\left(\frac{2\zeta\beta}{1-\beta^2}\right)$

Useful Trig ID: $A\sin\theta + B\cos\theta = G\sin(\theta + \eta)$ (4)

where: $G = \sqrt{A^2 + B^2}$

$\eta = \tan^{-1}\left(\frac{B}{A}\right)$ (5)

For our case: $A = 1$ $\quad B = \frac{c\bar{\omega}}{K}$

Then: $G = \sqrt{(1)^2 + \left(\frac{c\bar{\omega}}{K}\right)^2}$

$\eta = \tan^{-1}\left(\frac{c\bar{\omega}}{K}\right)$

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Realizing that: $\frac{c}{k} = \frac{2\zeta}{\omega}$

Then: $G = \sqrt{(1) + \left(\frac{2\zeta\bar{\omega}}{\omega}\right)^2}$ By defn $\beta = \frac{\bar{\omega}}{\omega}$

$$G = \sqrt{1 + (2\zeta\beta)^2} \quad \& \quad \eta = \tan^{-1}(2\zeta\beta)$$

Substituting, equation (3) becomes:

$$z_{ss}(t) = (0.03)\left(\frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)\left[G \sin(\bar{\omega}t \underbrace{-\phi + \eta}_{\gamma})\right]$$

$$= 0.03\left(\frac{\sqrt{1 + (2\zeta\beta)^2}}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right) \sin(\bar{\omega}t + \gamma) \qquad (6)$$

where: $\gamma = -\phi + \eta = \eta - \phi$

$$\tan(\gamma) = \tan(\eta - \phi) = \frac{\tan\eta - \tan\phi}{1 + \tan\eta \tan\phi} \qquad \text{(Trig ID)}$$

$$\tan(\gamma) = \frac{2\zeta\beta - \left(2\zeta\beta/1-\beta^2\right)}{1 + (2\zeta\beta)\left(\frac{2\zeta\beta}{1-\beta^2}\right)}$$

$$\tan(\gamma) = \frac{-2\zeta\beta^3}{(1-\beta^2) + (2\zeta\beta)^2} \qquad (7)$$

From comparing (6) w/ (2), then

$$X = \underbrace{0.03}_{y_0}\underbrace{\left(\frac{\sqrt{1 + (2\zeta\beta)^2}}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}\right)}_{R_d}$$

Want to keep $X < 0.12\,m$

$$y_0 R_d < 0.12\,m$$

$$(0.03)\,R_d < 0.12\,m$$

$$R_d < 4$$

Squaring both sides:

$$R_d^2 < 16$$

$$\frac{1+(2\zeta\beta)^2}{(1-\beta^2)^2+(2\zeta\beta)^2} < 16$$

For our case, want condition when $\beta = 1$. So:

$$\frac{1+(2\zeta)^2}{(2\zeta)^2} < 16$$

$$1 + 4\zeta^2 < 16(4\zeta^2)$$

$$1 + 4\zeta^2 < 64\zeta^2$$

$$1 < 60\zeta^2$$

$$\zeta^2 > 0.016667$$

$$\underline{\underline{\zeta > 0.129}}$$

To determine damp coef: $\zeta = \frac{c}{c_{cr}}$

$$c = \zeta c_{cr} = \zeta(2m\omega)$$

$$c = (0.129)[2(750\,kg)(31.42\,rad/s)]$$

$$\underline{\underline{c = 6084\ N\text{-}s/m}}$$