Problem 2: (Based on Old Exam Problem) For the system shown below, all surfaces are frictionless and the cables inextensible. A motor located on top of M_1 can be represented by a small mass of magnitude m eccentrically placed at a distance e. Given system properties of:

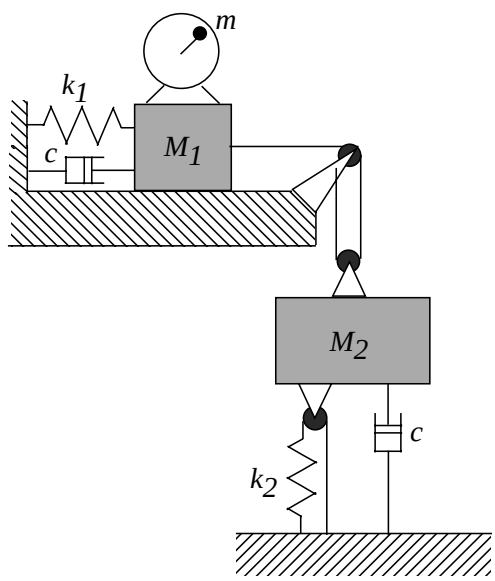
W_1 = 25 lbs, W_2 = 10 lbs, w = 2 lbs, k_1 = 2 lb/in, k_2 = 4 lb/in, e = 0.001 in

a) Draw the free-body diagram for all masses in the system. Indicate on the drawing what direction you are assuming as positive motion, and clearly state whether measuring motion from equilibrium or undeformed spring position.

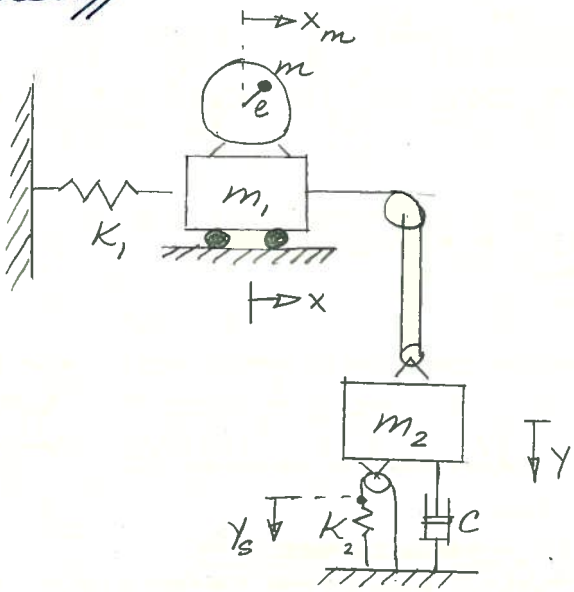
b) Formulate the equation of motion for the system based on the horizontal motion of M_1. Leave equation in symbolic form.

c) What is the undamped natural frequency? What is the damping ratio? Leave expressions in symbolic form.

d) What should the damping coefficient, c, for a single damper be so that the amplitude of dynamic motion of M_1 in steady-state is 5 inches in the *worst case* excitation? Hint: what ratio of forcing frequency to natural frequency creates the worst-case scenario?



Problem //



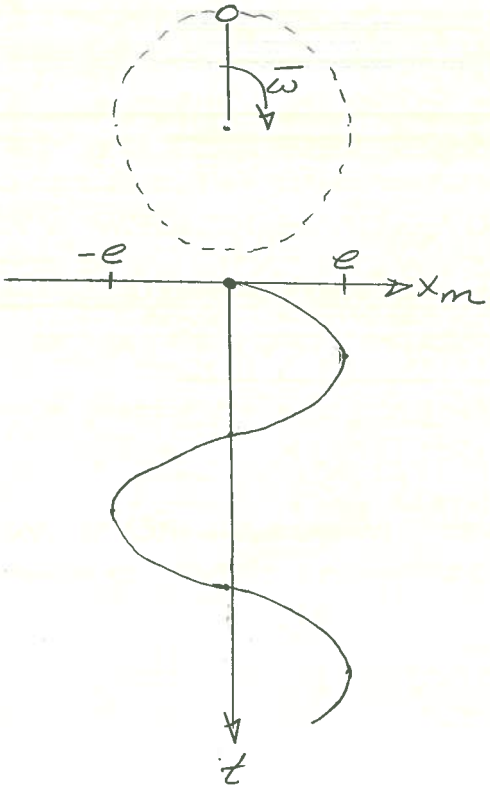
Note: rotation of unbalanced mass is a given/input to the system.

Assume system is pre-stressed so the cables never go slack

x = TOTAL horizontal motion of m_1 from s.e.

y = TOTAL vertical motion of m_2 from s.e.

x_m = horizontal motion of unbalanced mass RELATIVE to position of m_1.



Assume unbalanced mass:

- starts vert. upright
- rotates CW (then motion initially in + direction)

then as m rotates, the motion in the x direction is found by projecting it's place down onto the x_m axis.

In this case:

$$x_m = e \sin(\bar{\omega} t)$$

$$\ddot{x}_m = -e\bar{\omega}^2 \sin(\bar{\omega} t)$$

The equation for x_m may be a sin or cos function depending on your assumption of initial position & direction of rotation

<u>Note</u>: easiest to assume that rotation will cause mass to initially move in positive direction

<u>DOF</u> = x

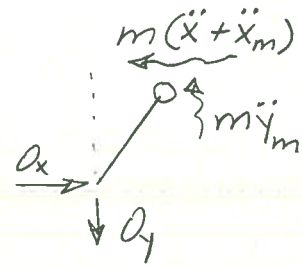
<u>Kinematics</u>:

$$2y = x$$

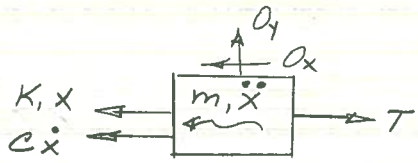
$$y_s = 2y = 2\left(\frac{x}{2}\right) = x$$

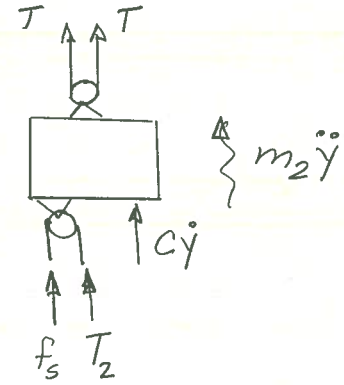
a)

<u>Free Body Diagrams</u> - Dynamic Only Forces



<u>Note</u>: draw FBC of masses deformed/moved in your assumed POSITIVE direction (not how you think it moves)





$$f_s = K_2(2y)$$

$$T_2 = f_s = (2K)y$$

b) Apply equilibrium

① $m_1\ddot{x} + k_1 x + c\dot{x} + O_x - T = 0$

② $m(\ddot{x} + \ddot{x}_m) - O_x = 0$

③ $m_2\ddot{y} + 2(2k_2 y) + c\dot{y} + 2T = 0$

Combining ① & ②

$$m_1\ddot{x} + m(\ddot{x} + \ddot{x}_m) + k_1 x + c\dot{x} - T = 0$$

④ $$(m_1 + m)\ddot{x} + k_1 x + c\dot{x} - T = -m\ddot{x}_m$$

Combine ③ & ④ to eliminate T

$$2(m_1 + m)\ddot{x} + m_2\ddot{y} + (2k_1)x + 4k_2 y + 2c\dot{x} + c\dot{y} = -2m\ddot{x}_m$$

Subs. kinematic constraint

$$2(m_1 + m)\ddot{x} + m_2\left(\frac{\ddot{x}}{2}\right) + (2k_1)x + (4k_2)\left(\frac{x}{2}\right) + 2c\dot{x} + c\left(\frac{\dot{x}}{2}\right) = -2m$$

$$(4m_1 + 4m + m_2)\ddot{x} + (4k_1 + 4k_2)x + (5c)\dot{x} = (-4m)\ddot{x}_m$$

Substituting for $\ddot{x}_m$:

$$\underbrace{(4m_1 + 4m + m_2)}_{m^*}\ddot{x} + \underbrace{(4k_1 + 4k_2)}_{k^*}x + \underbrace{(5c)}_{c^*}\dot{x} = \underbrace{(4me\bar{\omega}^2)}_{f_0}\sin(\bar{\omega}t)$$

So EOM is of the form:

$$m^*\ddot{x} + k^* x + c^*\dot{x} = f_0 \sin(\bar{\omega}t)$$

∴ Basically harmonic forced-vibration

c)

Undamped natural frequency

$$\omega = \sqrt{\frac{K^*}{m^*}} = \sqrt{\frac{4K_1 + 4K_2}{4m_1 + 4m + m_2}}$$

$$\zeta = \frac{c^*}{C_{cr}} = \frac{c^*}{2m^*\omega}$$

$$= \frac{5c}{2(4m_1 + 4m + m_2)}\sqrt{\frac{4m_1 + 4m + m_2}{4K_1 + 4K_2}}$$

d) $c = ?$ so that $\rho \leq 5$ inches when $\beta = 1$

- 1st find numerical values

$$m_1 = \frac{25\ \text{lbs}}{386.4\ \text{in/s}^2} = 0.06470\ \text{lb-s}^2/\text{in}$$

$$m_2 = \frac{10\ \text{lbs}}{386.4\ \text{in/s}^2} = 0.02588\ \text{lb-s}^2/\text{in}$$

$$m = \frac{2\ \text{lbs}}{386.4\ \text{in/s}^2} = 5.176 \times 10^{-3}\ \text{lb-s}^2/\text{in}$$

$$m^* = 4m_1 + 4m + m_2 = 0.3054\ \text{lb-s}^2/\text{in}$$

$$K^* = 4K_1 + 4K_2 = 4(2) + 4(4) = 24\ \text{lb/in}$$

$$\omega = \sqrt{\frac{K^*}{m^*}} = \sqrt{78.585} = 8.865\ \text{rad/sec}$$

Since want $\beta = 1 = \bar{\omega}/\omega$

Then $\bar{\omega} = \omega = 8.865\ \text{rad/sec}$

$$f_o = 4me\bar{\omega}^2$$
$$= 4(5.176\times10^{-3})(0.001)(8.865)^2$$
$$= 1.627\times10^{-3} \text{ lbs}$$

$$\Delta_{ST} = \frac{f_o}{K^*} = \left(1.627\times10^{-3}\right)/(24)$$

$$= 6.779\times10^{-5} \text{ inches}$$

Want: $\rho \le 5\text{in}$ So: $R_d \Delta_{ST} \le 5\text{in}$

$$R_d \le 5/\Delta_{ST}$$

$$R_d \le \frac{5}{6.779\times10^{-5}}$$

$$\left(\frac{1}{\sqrt{(1-\beta^2)^2+(2\zeta\beta)^2}} = R_d\right) \le 73755.4$$

$$73{,}755.4 = \frac{1}{\sqrt{(1-1^2)^2+(2\zeta(1))^2}}$$

$$73{,}7554\,(2\zeta) = 1$$

$$\zeta = 6.779\times10^{-6} \rightarrow \underbrace{6.779\times10^{-4}\,\%}_{\text{virtually no damp.}}$$

$$C_{cr} = 2m^*\omega = 2(0.3054)(8.865) = 5.415\ \frac{\text{lb-sec}}{\text{in}}$$

$$c^* = \zeta C_{cr} = (6.779\times10^{-6})(5.415) = 3.671\times10^{-5}$$

$$c = \frac{c^*}{5} = 7.341\times10^{-6}\ \frac{\text{lb-sec}}{\text{in}}$$